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Kaonic deuterium

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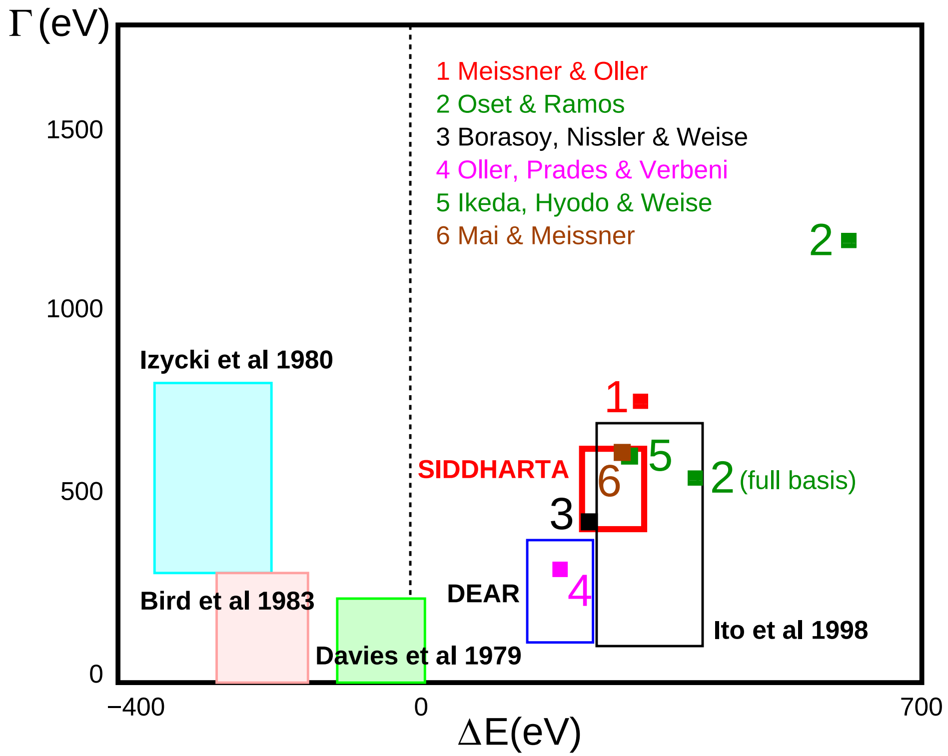
TU Munich, 26 October 2012

HISKP

Plan

- Experimental status
- Physics case
- Kaonic hydrogen
- Kaonic deuterium
  - *General remarks*
  - *Static approximation: re-summed formula and analysis of data*
  - *A systematic inclusion of the nucleon recoil*
- Conclusions & outlook

Experimental status



SIDDHARTA, kaonic hydrogen, arXiv:1201.4635 [nucl.ex]

$$\epsilon_{1s} = 283 \pm 36\text{(stat)} \pm 6\text{(syst) eV}\,, \quad \Gamma_{1s} = 541 \pm 89\text{(stat)} \pm 22\text{(syst) eV}$$

+ measurements of the kaonic deuterium, ^3He and ^4He.

Theoretical background: unitary baryon ChPT

The Lagrangians:

$$\mathcal{L} = \mathcal{L}_1 + \mathcal{L}_2 + \cdots$$

$$\begin{aligned}
\mathcal{L}_1 &= \langle i\bar{B}\gamma^\mu[D_\mu, B]\rangle - m_0\langle\bar{B}B\rangle + \frac{D}{2}\langle\bar{B}\gamma^\mu\gamma^5\{u_\mu, B\}\rangle \\
&+ \frac{F}{2}\langle\bar{B}\gamma^\mu\gamma^5[u_\mu, B]\rangle \\
\mathcal{L}_2 &= b_0\langle\bar{B}B\rangle\langle\chi_+\rangle + b_D\langle\bar{B}\{\chi_+, B\}\rangle + b_F\langle\bar{B}[\chi_+, B]\rangle \\
&+ b_1\langle\bar{B}[u_\mu, [u^\mu, B]]\rangle + b_2\langle\bar{B}\{u_\mu, \{u^\mu, B\}\}\rangle \\
&+ b_3\langle\bar{B}\{u_\mu, [u^\mu, B]\}\rangle + b_4\langle\bar{B}B\rangle\langle u_\mu u^\mu\rangle + \cdots \\
\cdots
\end{aligned}$$

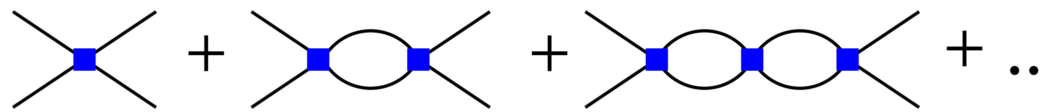
. . . plus unitarization of the multi-channel scattering amplitudes

The role of the unitarity

Iteration of the potential evaluated in ChPT:

N. Kaiser, P. B. Siegel and W. Weise, NPA 594 (1995) 325

$$T = V + VG_0T =$$



Recent calculations with the NLO potential:

Y. Ikeda, T. Hyodo and W. Weise, PLB 706 (2011) 63, NPA 881 (2012) 98

M. Mai and U.-G. Meißner, arXiv:1202.2030

Input:

The scattering data for $\bar{K}p \to \bar{K}p,\ \pi\Sigma,\ \pi\Lambda$
plus three threshold values γ, R_c, R_n

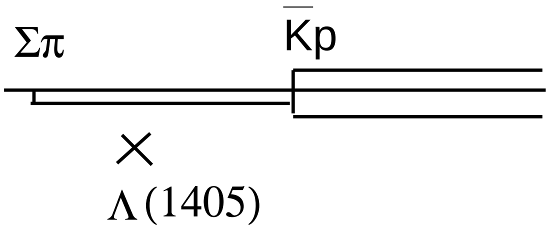
The kaonic hydrogen energy shift and width

Output:

Accurate values of the $\bar{K}N$ scattering lengths a_0, a_1

What can one learn on the lattice?

V. Bernard, M. Lage, U.-G. Meißner and AR,
PLB 681 (2009) 439



- Two channels: “heavy”=1, “light”=2
- A resonance in the vicinity of “heavy” threshold $s = s_t$
- Below “heavy” threshold $s = s_t$, the inelasticity is small: $\eta \simeq 1$

$$
\begin{aligned}
T_{11} &= H_{11} + H_{11} i q_1 T_{11} + H_{12} i q_2 T_{21} \\
T_{21} &= H_{21} + H_{21} i q_1 T_{11} + H_{22} i q_2 T_{21}
\end{aligned}
$$

Complex scattering length (s_t denotes “heavy” threshold):

$$
a_{11} = H_{11}(s_t) + \frac{i q_2(s_t)(H_{12}(s_t))^2}{1 - i q_2(s_t) H_{22}(s_t)}
$$

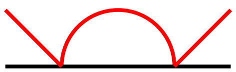
Resonance poles are determined from secular equation:

$$
1 - i q_1 H_{11} - i q_2 H_{22} - q_1 q_2 (H_{11} H_{22} - H_{12}^2) = 0
$$

Energy spectrum in a finite volume

Finite volume: $\displaystyle \int \frac{d^3\mathbf{k}}{(2\pi)^3}(\cdots) \to \frac{1}{L^3}\sum_{\mathbf{k}}(\cdots), \quad \mathbf{k} = \frac{2\pi\mathbf{n}}{L}, \ \mathbf{n} \in Z^3$

Loop function goes into Lüscher zeta-function



$$: \qquad iq_i \to \mathcal{J}_i = \frac{2}{\sqrt{\pi}L} Z_{00}(1; k_i^2)\,, \quad k_i = \frac{Lq_i}{2\pi}\,, \quad i = 1, 2$$

The location of the energy levels is determined by secular equation

$$1 - \mathcal{J}_1 H_{11} - \mathcal{J}_2 H_{22} + \mathcal{J}_1 \mathcal{J}_2 (H_{11} H_{22} - H_{12}^2) = 0$$

$\hookrightarrow$ $\sqrt{s_n} = E_n = E_n(H_{ij}^{(\alpha)}; L)\,, \qquad i, j = 1, 2, \quad \alpha = 0, 1, 2, \cdots$

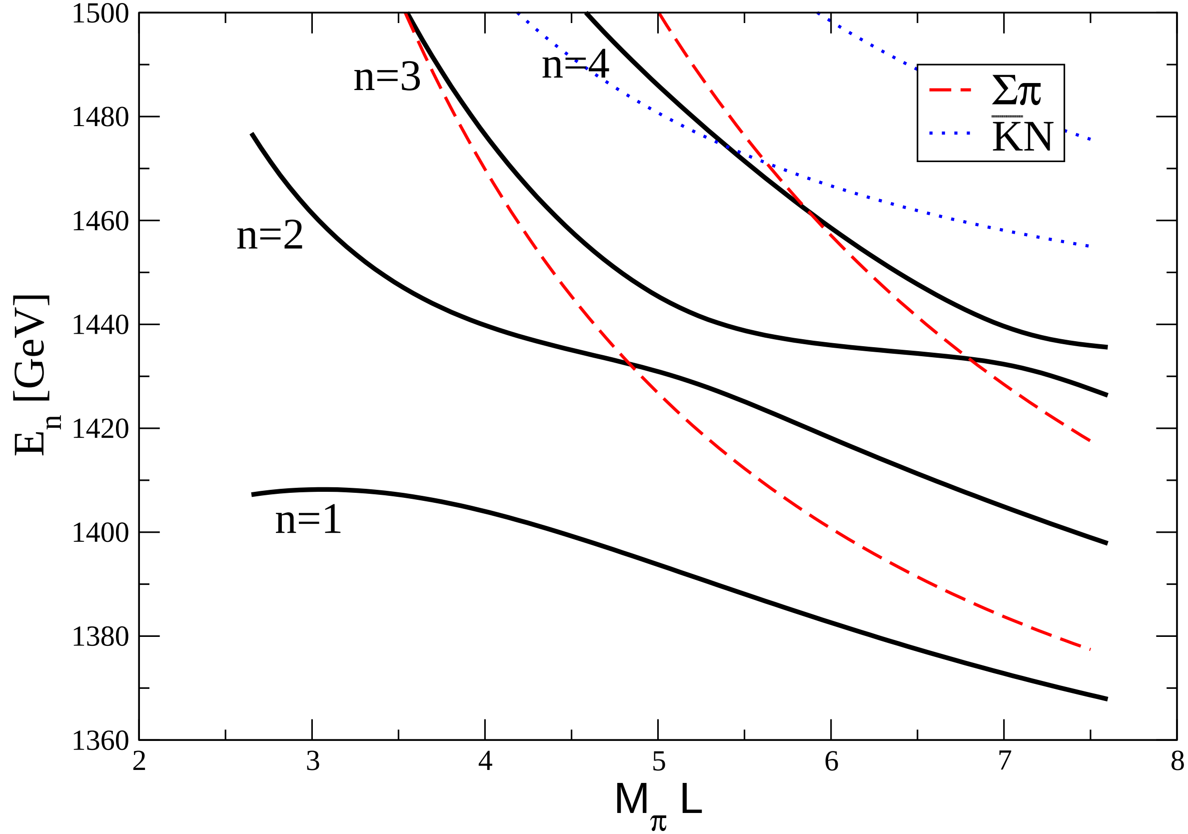
$\hookrightarrow$ Determine $H_{ij}^{(\alpha)}$ from the fit to $E_n = E_n(H_{ij}^{(\alpha)}; L)$

$\hookrightarrow$ Express the scattering lengths & resonance poles through $H_{ij}^{(\alpha)}$

An example of the energy levels: $\bar{K}N$, $\Sigma\pi$ system

$\bar{K}N$ threshold $\longrightarrow$

$\Lambda\,(1405)$ $\longrightarrow$



The physics background

Unitary ChPT:

- Unitary ChPT can in principle predict the results of the hadronic atom measurements
- Significant restrictions, if hadronic atom data are used as an input; implications for the K^- interactions in the medium

The lattice QCD can yield:

- The complex $\bar{K}N$ scattering lengths
- σ-terms and the strangeness content of the baryons

R. Horsley *et al*, PRD 84 (2011) 074507

KAONNIS/SIDDHARTA experiment:

- Extract scattering lengths a_0, a_1 from the experiments on kaonic hydrogen and deuterium
- Compare with unitary ChPT and lattice results

Energy shift of the kaonic hydrogen

U.-G. Meißner, U. Raha and AR, EPJC 35 (2004) 349

$$\Delta E_{1s} - i\,\frac{\Gamma_{1s}}{2} = -2\alpha^3\mu^2 a_p \left(1 - \underbrace{2\mu\alpha(\ln\alpha - 1)a_p}_{\text{Coulomb } \simeq 10\%}\right) + \cdots$$

Large corrections due to the unitary cusp:

$$a_p = \frac{\frac{1}{2}\,(a_0 + a_1) + q_0 a_0 a_1}{1 + \frac{q_0}{2}\,(a_0 + a_1)}\,, \qquad q_0 = (2\mu_0\Delta\mathcal{M})^{1/2}$$

R.H. Dalitz and S.F. Tuan, Ann. Phys. **10** (1960) 307

Improvement: re-summing Coulomb corrections

$$1 - 2\mu\alpha(\ln\alpha - 1)a_p \to (1 + 2\mu\alpha(\ln\alpha - 1)a_p)^{-1}$$

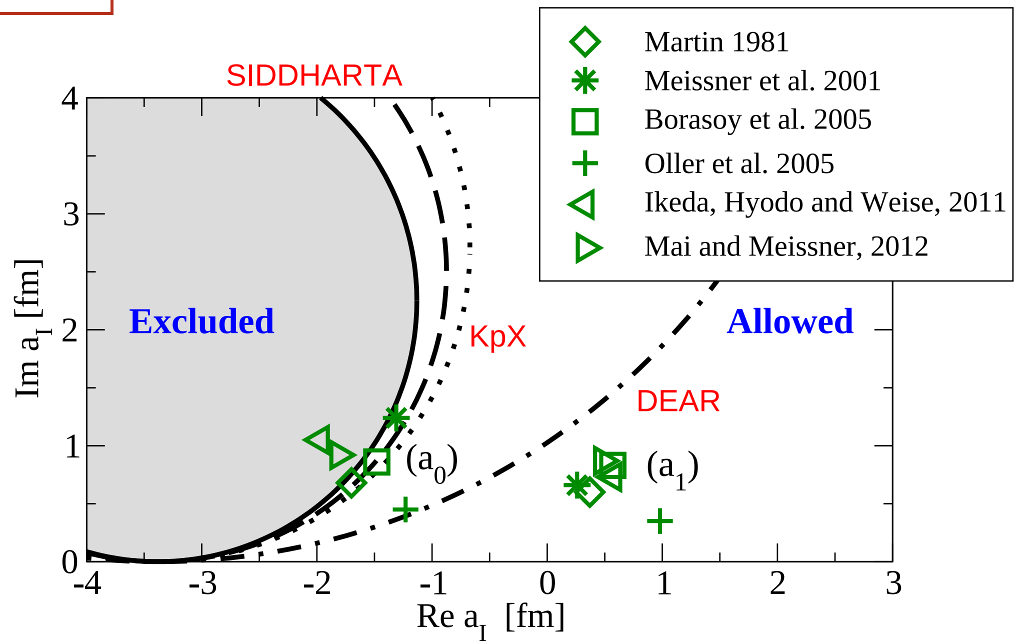
A. Cieply and J. Smejkal, arXiv:0710.3600 [nucl-th], potential model

Constraints imposed by kaonic hydrogen data

$$a_0 + a_1 + \frac{2q_0}{1 - q_0 a_p}\, a_0 a_1 - \frac{2a_p}{1 - q_0 a_p} = 0\,, \qquad \mathrm{Im}\, a_I \geq 0 \quad \text{(unitarity)}$$

U.-G. Meißner, U. Raha and AR, EPJC 47 (2006) 473

Errors not shown



M. Döring and U.-G. Meißner, PLB 704 (2011) 663

Extracting scatting lengths a_0 and a_1

- a_0 and a_1 separately can not be extracted from KH data alone, needs $KH + Kd \quad \Rightarrow \quad$ 4 real quantities

$$E_{1s}^d = \frac{1}{2}\,\alpha\mu_d^2 \simeq 10.4\ \text{keV}\,, \quad \Delta E_{1s}^d,\ \Gamma_{1s}^d \quad \text{of order of } 1\ \text{keV}$$

$$\Delta E_{1s}^d - i\,\frac{\Gamma_{1s}^d}{2} = -2\alpha^3\mu_d^2 A_{\bar{K}d}\left(1 - 2\mu\alpha(\ln\alpha - 1)A_{\bar{K}d}\right) + \cdots$$

$a_p,\ A_{\bar{K}d} \Rightarrow S$-wave $\bar{K}N$ scattering lengths a_0 and a_1

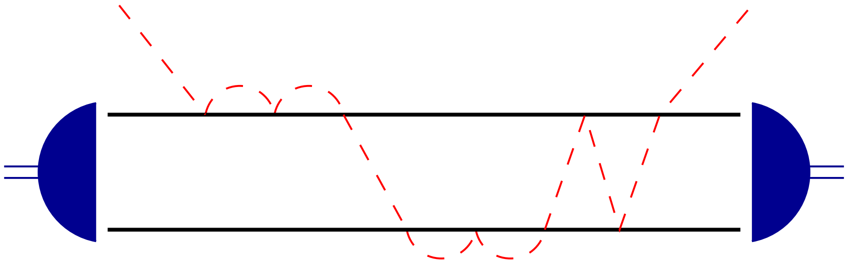
- Explicitly relating $A_{\bar{K}d}$ to a_0 and a_1 through multiple-scattering series is needed
- Isospin-breaking corrections should be taken into account

Do the systematic uncertainties allow for an extraction of a_0 and a_1?

Multiple-scattering series in FCA

Infinitely heavy nucleon,

Kaon hopping between nucleons



$$\left(1+\frac{M_K}{M_d}\right)A_{\bar{K}d}=\int d^3\mathbf{r}|\Psi(\mathbf{r})|^2\mathcal{A}_{\bar{K}d}(r)\,, \qquad q=(2M_K\Delta\mathcal{M})^{1/2}$$

$$\mathcal{A}_{\bar{K}d}(r)=\frac{\tilde{t}_p+\tilde{t}_n+(2\tilde{t}_p\tilde{t}_n-b_x^2)/r-2b_x^2\tilde{t}_n/r^2}{1-\tilde{t}_p\tilde{t}_n/r^2+b_x^2\tilde{t}_n/r^3}\,,\quad b_x^2=\frac{\mathrm{e}^{-qr}\,\tilde{t}_x^2}{1+\mathrm{e}^{-qr}\tilde{t}_n^0/r}$$

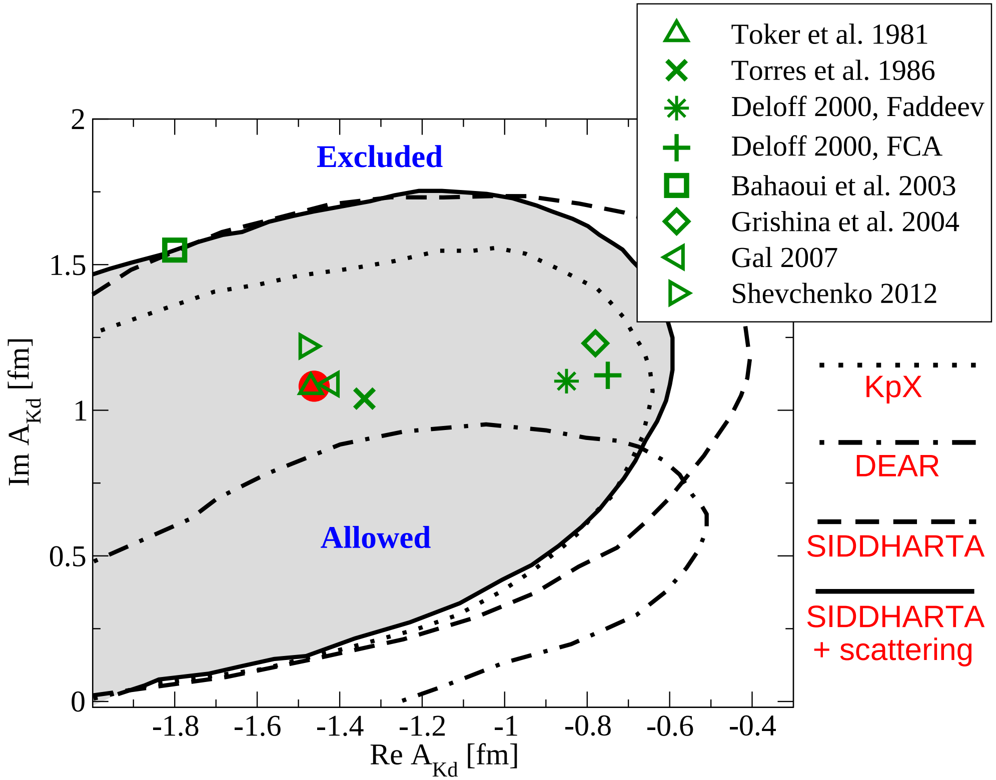
R. Chand and R. H. Dalitz, Ann. Phys. 20 (1962) 1

S. Kamalov, E. Oset and A. Ramos, NPA 690 (2001) 494

U.-G. Meißner, U. Raha and AR, EPJC 47 (2006) 473

$\bar{K}H + \bar{K}d$ simultaneous data analysis

- Experimental input: SIDDHARTA result for the kaonic hydrogen
- Theoretical input: Fixed Center Approximation ($m_N \to \infty$)



M. Döring and U.-G. Meißner, PLB 704 (2011) 663

see also U.-G. Meißner, U. Raha and AR, EPJC 47 (2006) 473

Issues to be addressed

- Status of the multiple scattering expansion
- Systematic uncertainty, e.g., due to the 3-body force
- Isospin-breaking in the $\bar{K}d$ scattering length
- Does the close-by $\Lambda(1405)$ lead to a significant contribution, e.g., through large effective-range term?
- Taking into account the nucleon recoil:

$\hookrightarrow$ In the potential models, the re-summed multiple scattering series usually reproduces the result of full Faddeev calculations to a good accuracy

Why is the FCA so accurate even for $M_K/m_N \simeq 0.5$?

EFT approach to $\bar{K}d$ scattering

V. Baru, E. Epelbaum and AR, EPJA 42 (2009) 111

- Different momentum scales $\longrightarrow$ multiple-scattering expansion

$NN,\ \bar{K}NN$	:	*one-pion exchange*
$\bar{K}N$	:	*two-pion exchange*

- The convergence of the series is controlled by $a \cdot \langle \frac{1}{r} \rangle \simeq 1$, where $\langle \frac{1}{r} \rangle \simeq 0.5$ fm^{-1}. S-wave scattering lengths are large due to the presence of the subthreshold $\Lambda(1405)$ resonance

$\longrightarrow$ Re-summation (is done in the FCA)

- Exact solution of Faddeev equations:

$\longrightarrow$ Retardation effects moderate albeit $\xi = M_K/m_N \simeq 0.5$

$\longrightarrow$ To be understood in a systematic approach based on EFT

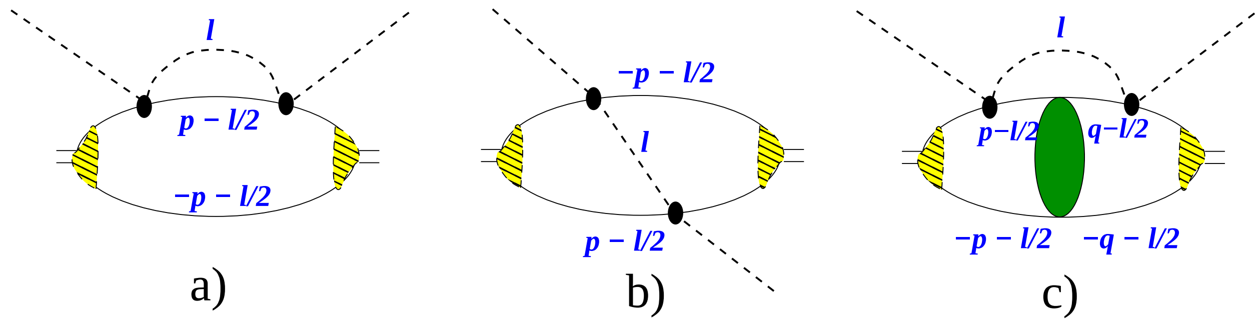
Non-relativistic effective Lagrangian

$$
\begin{aligned}
\mathcal{L} &= K^\dagger\left(i\partial_t - M_K + \frac{\triangle}{2M_K} + \cdots\right)K + c\,\psi^\dagger K^\dagger K\psi \\
&- \frac{1}{2}\,d\,\psi^\dagger(\triangle K^\dagger K + K^\dagger \triangle K)\psi + \cdots + \psi^\dagger\psi^\dagger K^\dagger\,\mathcal{V}_{\bar{\mathcal{K}}\mathcal{N}\mathcal{N}}\,K\psi\psi + \cdots \\
&+ \psi^\dagger\left(i\partial_t - m_N + \frac{\triangle}{2m_N} + \cdots\right)\psi + \psi^\dagger\psi^\dagger\,\mathcal{V}_{\mathcal{N}\mathcal{N}}\,\psi\psi
\end{aligned}
$$

- $c, d \cdots \quad \Leftrightarrow$ scattering length, effective radius...

Perturbative expansion $\Rightarrow$ multiple-scattering expansion

Retardation corrections: second order



$$A_{\bar{K}d}^{\text{doubl. scatt.}} = \frac{8\pi\mu_d M_K}{\mu^2}\,(R_a + R_b + R_c)$$

$$R_i = R_i^{\text{stat}} + \xi^{1/2} R_i^{(1)} + \xi R_i^{(2)} + \xi^{3/2} R_i^{(3)} + \cdots$$

$\hookrightarrow$ Calculate $R_i^{(1)}, R_i^{(2)}, \ldots$, using *uniform expansion* method

Uniform expansion method

R. E. Mohr *et al*, Ann. Phys. 321 (2006) 225

see also M. Beneke and V. A. Smirnov, NPB 522 (1998) 321: *Threshold expansion*

Low-momentum regime $\longrightarrow$ half-integer powers of ξ

$$\frac{\mathbf{l}^2}{2M_K} \sim \frac{\mathbf{p}^2}{2m_N} \quad \Rightarrow \quad \mathbf{l} \sim \sqrt{\xi}\mathbf{p}\,, \quad \mathbf{p} \sim \langle \frac{1}{r} \rangle$$

High-momentum regime $\longrightarrow$ integer powers of ξ

$$\mathbf{l} \sim \mathbf{p} \sim \langle \frac{1}{r} \rangle$$

Intermediate regime

$$\sqrt{\xi}\mathbf{p} \ll \mathbf{l} \ll \mathbf{p}$$

$\hookrightarrow$ Expand the integrand in Taylor series in each region separately

Cancellation of leading corrections

$$R = R^{\text{stat}} + \xi^{1/2} R^{(1)} + \xi R^{(2)} + \xi^{3/2} R^{(3)} + \cdots$$

see also: G. Fäldt, Phys. Scripta 16 (1977) 81; V. Baru *et al*, PLB 589 (2004) 118

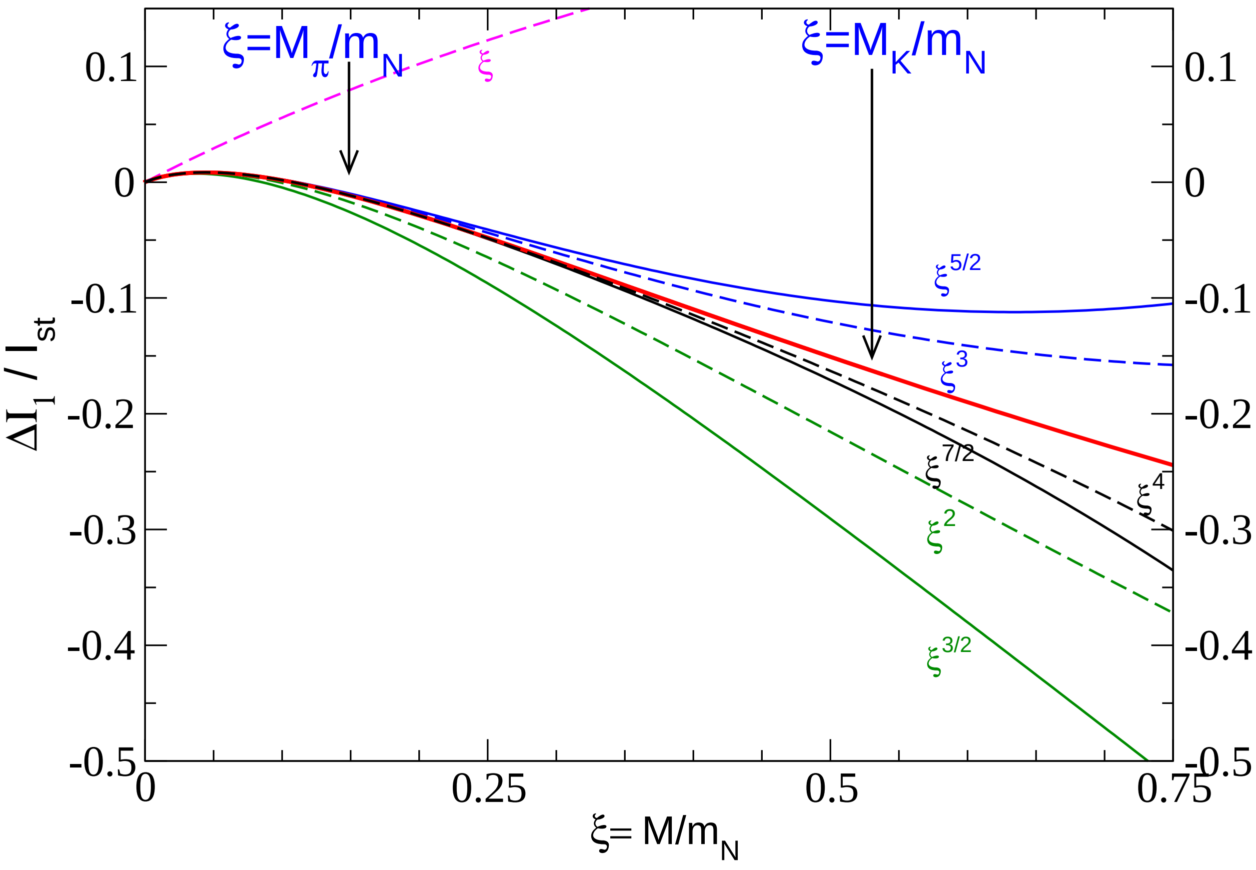
- Isospin-odd channel: Pauli-selection rules $\longrightarrow \quad R_-^{(1)} = 0$
- Isospin-even channel: at leading order in ξ,

$$R_+^{(1)} \sim \int \frac{d^3\mathbf{p}d^3\mathbf{q}d^3\mathbf{l}}{(2\pi)^6} \Psi(\mathbf{p}) \left(G_{NN}(\mathbf{p},\mathbf{q};E(\mathbf{l})) - \frac{\delta^3(\mathbf{p}-\mathbf{q})}{\mathbf{l}^2/2M_K} \right) \Psi(\mathbf{q})$$

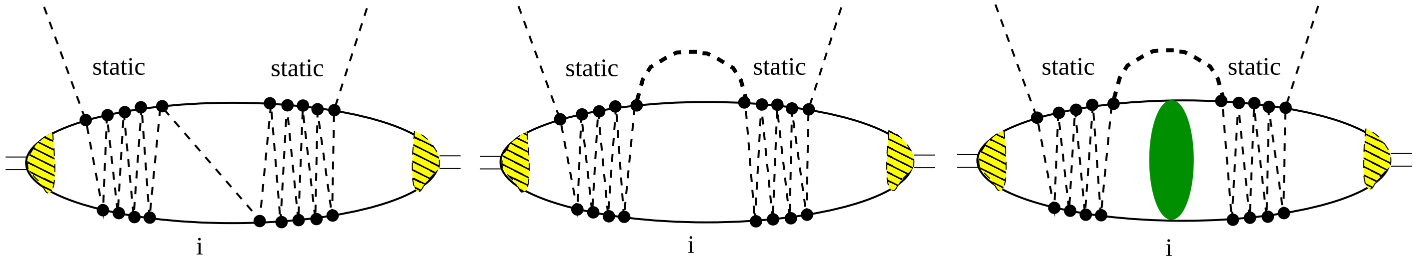
$\hookrightarrow$ Vanishes at leading order due to the orthogonality of the bound-state and continuum wave functions

Convergence of the expansion

Corrections to the isospin-odd amplitude
(the kinematical factor $(1+\xi)^{-1}$ is not expanded)



Multiple-scattering series: to be done



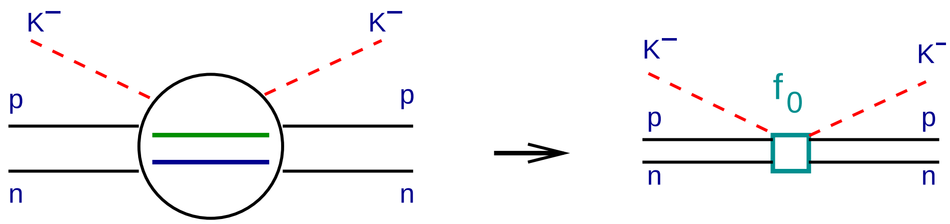
$\hookrightarrow$ Assume retardation correction perturbative, in contrast to static interactions

3-body force

Multiple-scattering series and the three-body force:

$$\int_0^\infty dr \frac{|\Psi(r)|^2}{1+a/r} = \int_0^\infty dr |\Psi(r)|^2 \left(1 - \frac{a}{r}\right) + \underbrace{|\Psi(0)|^2 \, aL}_{\text{3-body force}} + O(a^2)$$

Estimating the imaginary part from the two-nucleon absorption of K^-:



$$\begin{aligned} K^- + d &\rightarrow \Sigma^- + p \\ &\rightarrow \Sigma^0 + n \\ &\rightarrow \Lambda + n \end{aligned}$$

The total two-nucleon absorption rate $= (1.22 \pm 0.09)\%$

V.R. Veirs and R.A. Burnstein, PRD 1 (1970) 1883

$\hookrightarrow$ Three body forces: of order of a few percent

Conclusions & outlook

- In order to be able to analyze the forthcoming SIDDHARTA data on kaonic deuterium, a study of the multiple-scattering series is necessary. Potential model calculations alone do not suffice.
- EFT provides a natural tool to study the multiple-scattering expansion
- The retardation corrections are the central issue to address in the nearest future
  - $\Rightarrow$ *These are by far the largest source of uncertainty*
  - $\Rightarrow$ *Numerically, these are not very large (potential models)*
  - $\Rightarrow$ *It is a result of large cancellations (second order)*
  - $\Rightarrow$ *Need to carry out calculations non-perturbatively, summing up the multiple-scattering series*