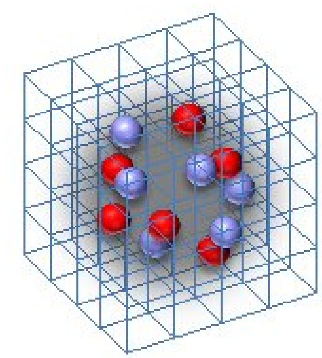
Status of Nuclear Lattice Simulations

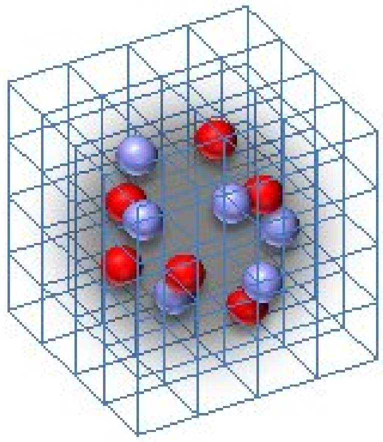
Ulf-G. Meißner, Univ. Bonn & FZ Jülich



NLEFT

Supported by DFG, SFB/TR-16 and by DFG, SFB/TR-110 and by EU, I3HP EPOS and by BMBF 06BN9006 and by HGF VIQCD VH-VI-417

- **Nuclear Lattice Effective Field Theory collaboration**



Evgeny Epelbaum (Bochum)

Hermann Krebs (Bochum)

Timo Lähde (Jülich)

Dean Lee (NC State)

Ulf-G. Meißner (Bonn/Jülich)

CONTENTS

Introduction: Effective Field Theory for Nuclear Physics

only a brief reminder $\rightarrow$ details in

E. Epelbaum, H.-W. Hammer, UGM, Rev. Mod. Phys. **81** (2009) 1773 [arXiv:0811.1338 [nucl-th]]

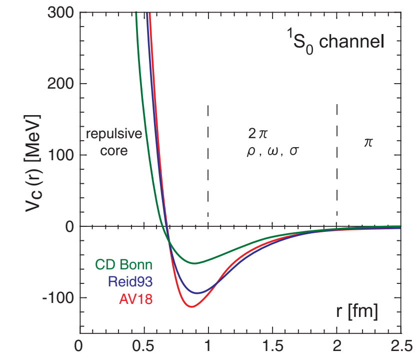
CHIRAL EFT FOR FEW-NUCLEON SYSTEMS

Gasser, Leutwyler, Weinberg, van Kolck, Epelbaum, Bernard, Kaiser, UGM, . . .

- Scales in nuclear physics:

  Natural: $\lambda_\pi = 1/M_\pi \simeq 1.5$ fm (Yukawa 1935)

  Unnatural: $|a_{np}(^1S_0)| = 23.8\,\mathrm{fm}\;,\;\; a_{np}(^3S_1) = 5.4\,\mathrm{fm} \gg 1/M_\pi$



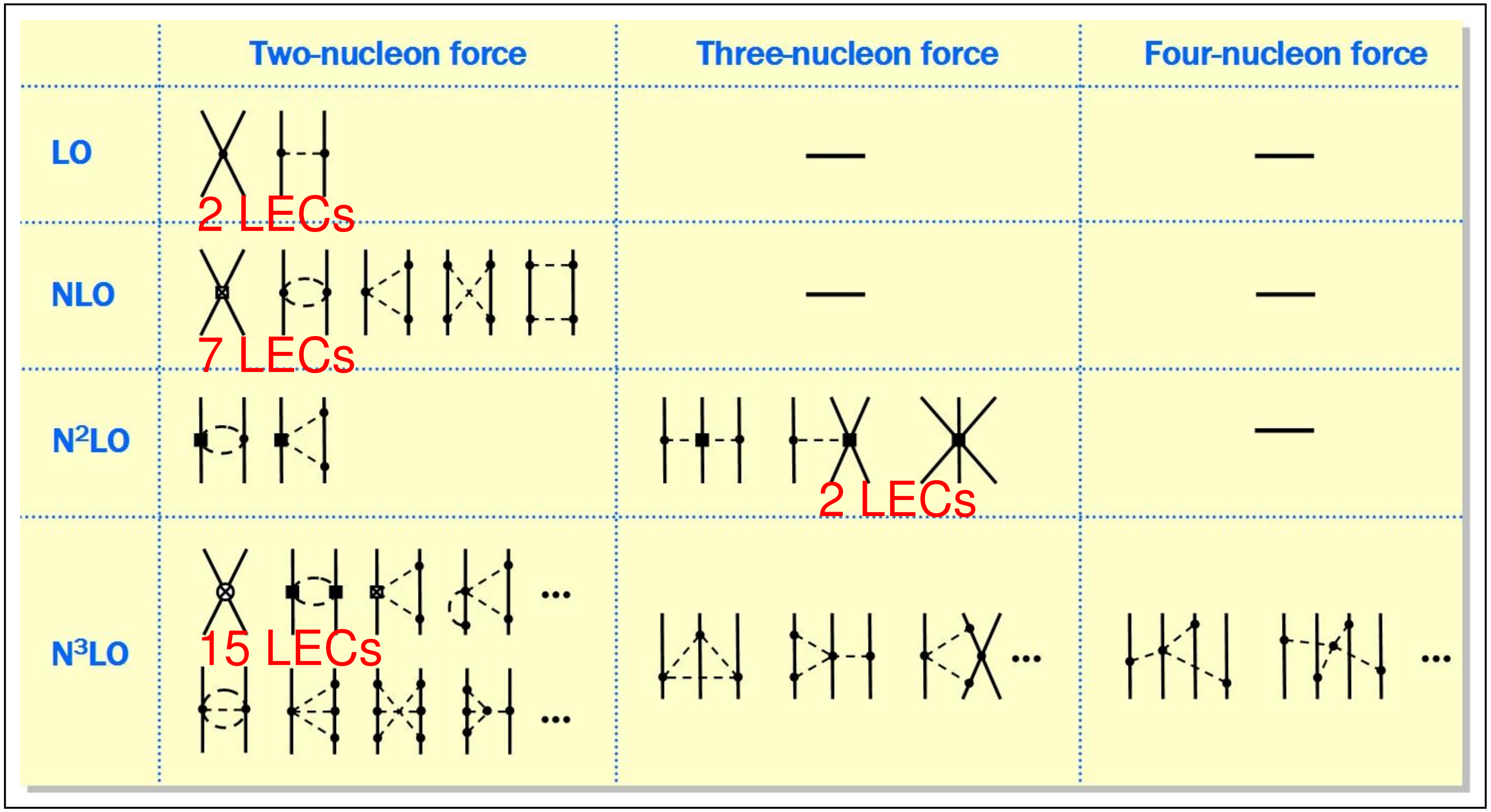
- this can be analyzed in a suitable EFT based on

$$\mathcal{L}_{\mathrm{QCD}} \to \mathcal{L}_{\mathrm{EFF}} = \mathcal{L}_{\pi\pi} + \mathcal{L}_{\pi N} + \mathcal{L}_{NN} + \ldots$$

- pion and pion-nucleon sectors are perturbative in Q/Λ_χ → chiral perturbation th'y
- $\mathcal{L}_{NN}$ collects short-distance contact terms, to be fitted
- NN interaction requires non-perturbative resummation

  → chirally expand $\mathrm{V}_{\mathrm{NN(N)}}$, use in regularized LS/FY equation

CHIRAL POTENTIAL and NUCLEAR FORCES



$\mathcal{O}((Q/\Lambda_\chi)^0)$

$\mathcal{O}((Q/\Lambda_\chi)^2)$

$\mathcal{O}((Q/\Lambda_\chi)^3)$

$\mathcal{O}((Q/\Lambda_\chi)^4)$

- explains naturally the observed hierarchy of nuclear forces
- MANY successfull tests in few-nucleon systems (continuum calc’s)

Nuclear lattice simulations – Formalism –

NUCLEAR LATTICE SIMULATIONS

Frank, Brockmann (1992), Koonin, Müller, Seki, van Kolck (2000) , Lee, Schäfer (2004), . . .
Borasoy, Krebs, Lee, UGM, Nucl. Phys. **A768** (2006) 179; Borasoy, Epelbaum, Krebs, Lee, UGM, Eur. Phys. J. **A31** (2007) 105

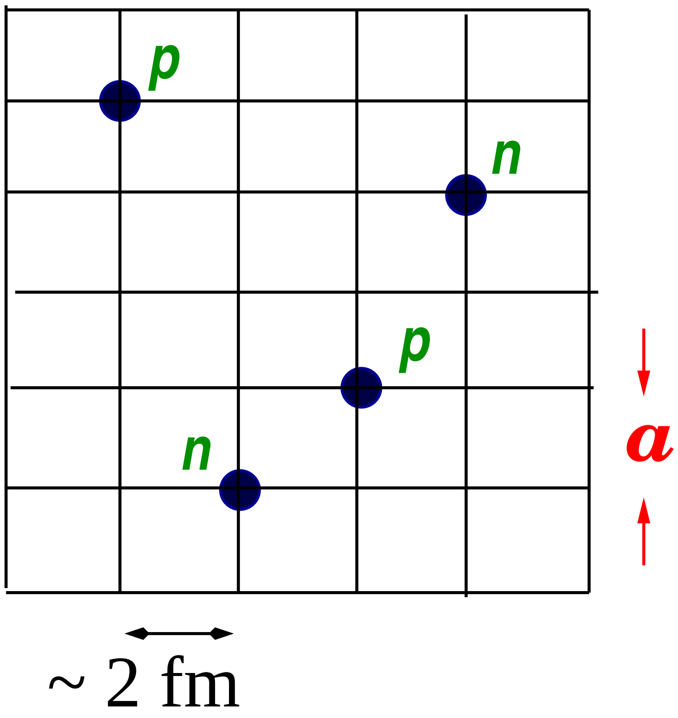
- *new method* to tackle the nuclear many-body problem

- discretize space-time $V = L_s \times L_s \times L_s \times L_t$:
  nucleons are point-like fields on the sites

- discretized chiral potential w/ pion exchanges
  and contact interactions

- typical lattice parameters

$$\Lambda = \frac{\pi}{a} \simeq 300\,\text{MeV [UV cutoff]}$$

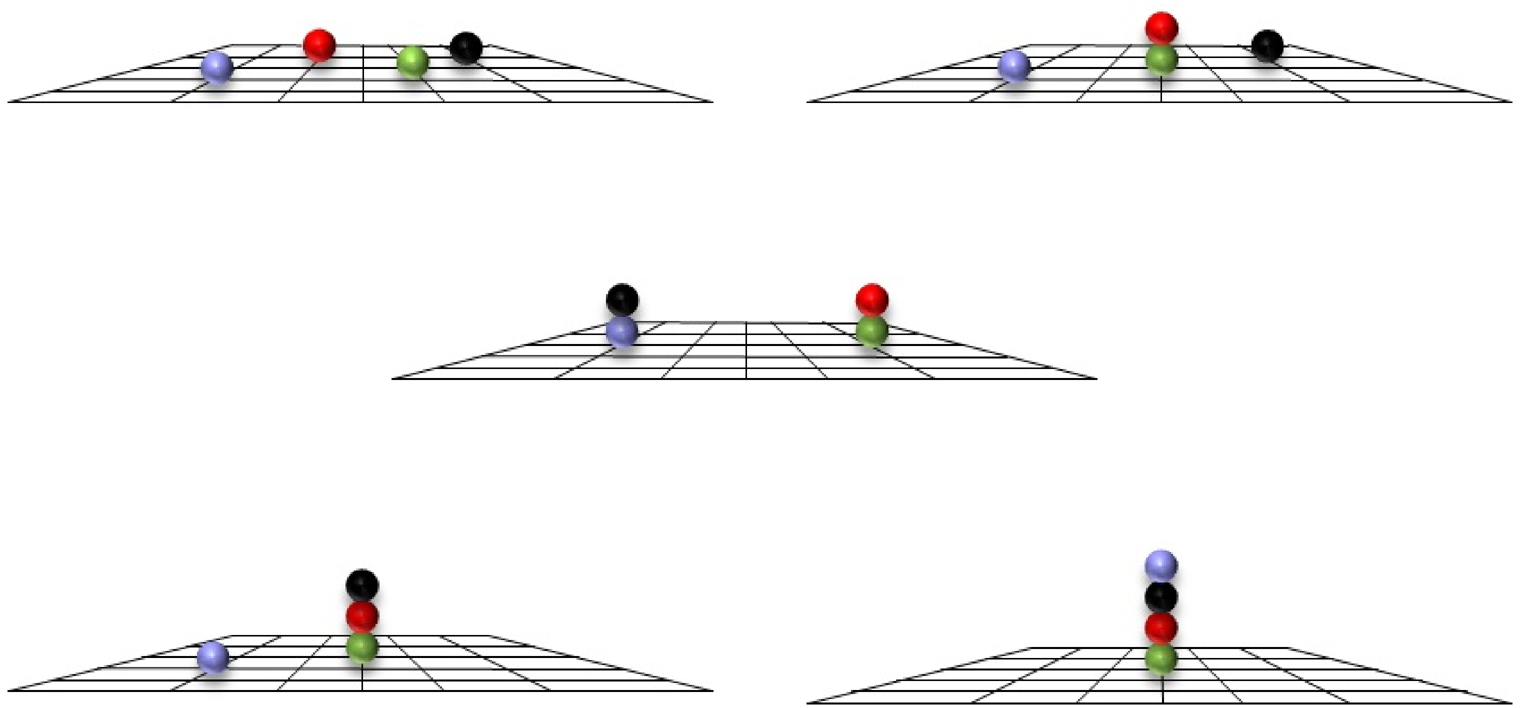


- strong suppression of sign oscillations due to approximate Wigner SU(4) symmetry

J. W. Chen, D. Lee and T. Schäfer, Phys. Rev. Lett. **93** (2004) 242302

- hybrid Monte Carlo & transfer matrix (similar to LQCD)

CONFIGURATIONS



⇒ all *possible* configurations are sampled

⇒ *clustering* emerges *naturally*

TRANSFER MATRIX METHOD

- Correlation–function for A nucleons: $Z_A(t) = \langle \Psi_A | \exp(-tH) | \Psi_A \rangle$

  with Ψ_A a Slater determinant for A free nucleons

Euclidean time

- Ground state energy from the time derivative of the correlator

$$E_A(t) = -\frac{d}{dt} \ln Z_A(t)$$

$\to$ ground state filtered out at large times: $E_A^0 = \lim_{t\to\infty} E_A(t)$

- Expectation value of any normal–ordered operator $\mathcal{O}$

$$Z_A^{\mathcal{O}} = \langle \Psi_A | \exp(-tH/2)\, \mathcal{O}\, \exp(-tH/2)\, | \Psi_A \rangle$$

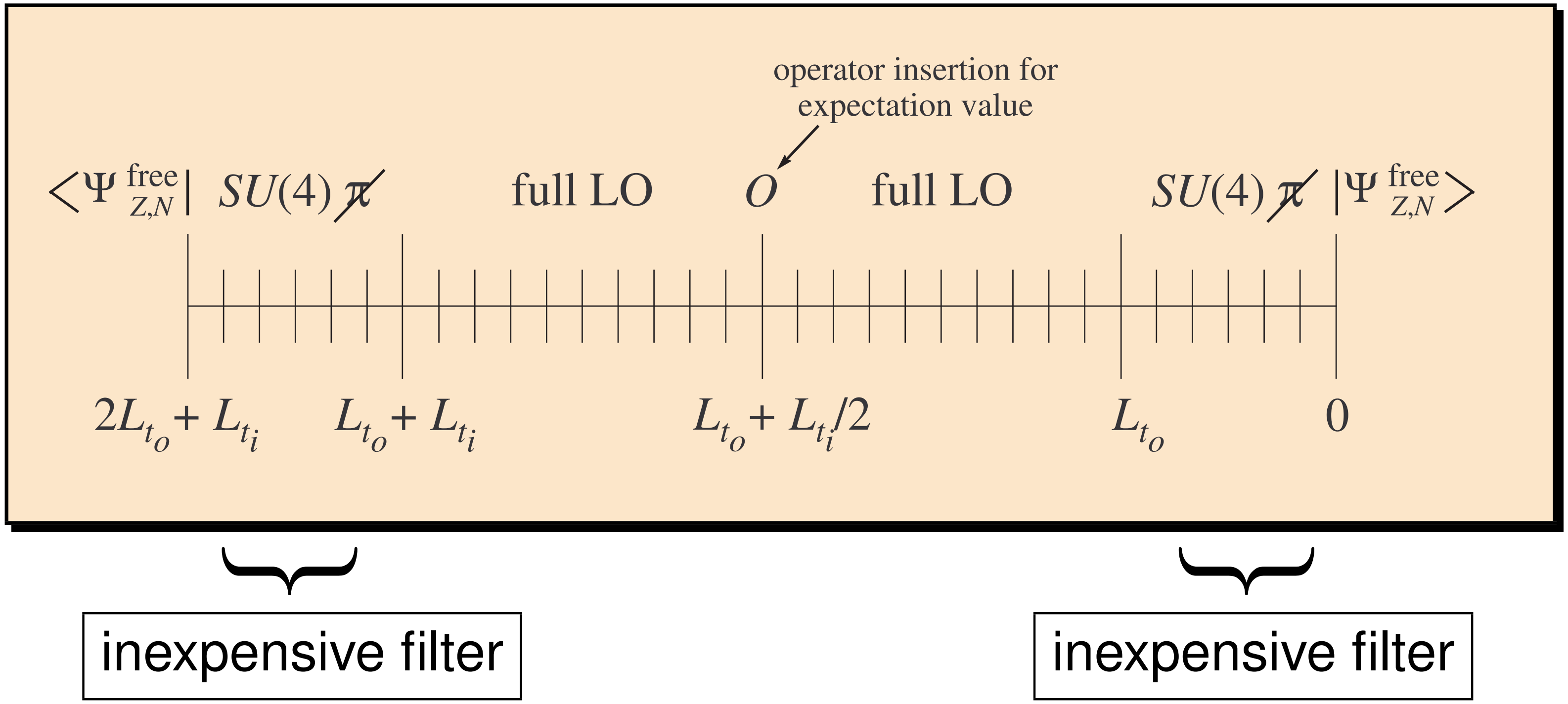
$$\lim_{t\to\infty} \frac{Z_A^{\mathcal{O}}(t)}{Z_A(t)} = \langle \Psi_A | \mathcal{O} \, | \Psi_A \rangle$$

TRANSFER MATRIX CALCULATION

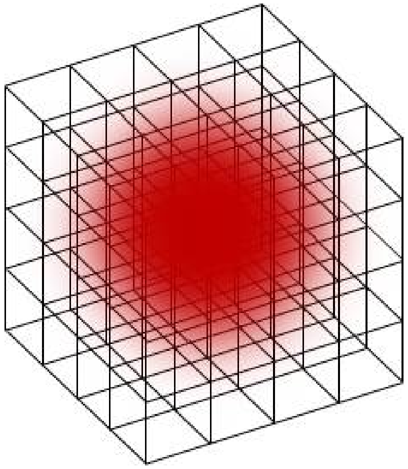
- Expectation value of any normal–ordered operator $\mathcal{O}$

$$\langle \Psi_A | \mathcal{O} | \Psi_A \rangle = \lim_{t\to\infty} \frac{\langle \Psi_A | \exp(-tH/2)\, \mathcal{O}\, \exp(-tH/2) | \Psi_A \rangle}{\langle \Psi_A | \exp(-tH) | \Psi_A \rangle}$$

- Anatomy of the transfer matrix



PROJECTION MONTE CARLO TECHNIQUE



- General wave function:

$$\psi_j(\vec{n})\ ,\quad j=1,\ldots,A$$

- States with well-defined momentum:

$$L^{-3/2}\sum_{\vec{m}}\psi_j(\vec{n}+\vec{m})\exp(i\vec{P}\cdot\vec{m})\ ,\quad j=1,\ldots,A$$

- Insert clusters of nucleons at initial/final states (spread over some time interval)
  $\to$ allows for all type of wave functions (shell model, clusters, ...)
  $\to$ removes directional bias

shell-model type

$$\psi_j(\vec{n})=\exp[-c\vec{n}^2]$$
$$\psi_j'(\vec{n})=n_x\exp[-c\vec{n}^2]$$
$$\psi_j''(\vec{n})=n_y\exp[-c\vec{n}^2]$$
$$\psi_j'''(\vec{n})=n_z\exp[-c\vec{n}^2]$$

cluster type

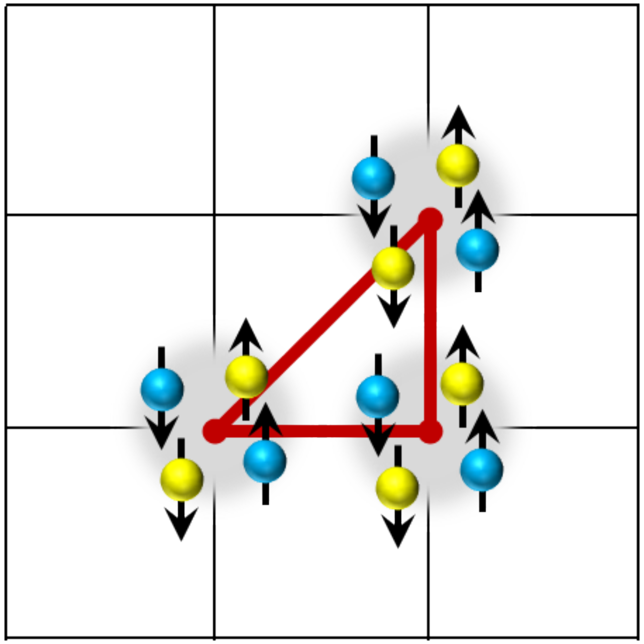
$$\psi_j(\vec{n})=\exp[-c(\vec{n}-\vec{m})^2]$$
$$\psi_j'(\vec{n})=\exp[-c(\vec{n}-\vec{m}')^2]$$
$$\psi_j''(\vec{n})=\exp[-c(\vec{n}-\vec{m}'')^2]$$
$$\psi_j'''(\vec{n})=\exp[-c(\vec{n}-\vec{m}''')^2]$$

- shell-model w.f.s do not have enough 4N correlations $\sim\langle(N^\dagger N)^2\rangle$

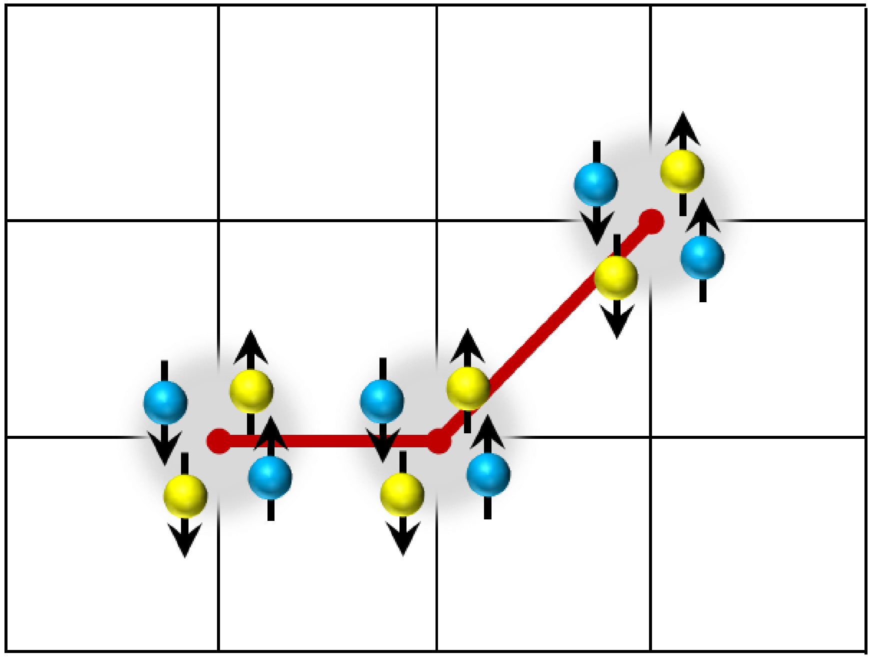
PROJECTION MONTE CARLO TECHNIQUE II

- Example: two basic configurations in the spectrum of ^{12}C $(a = 1.97\,\text{fm})$

compact triangle config.
12 rotational orientations



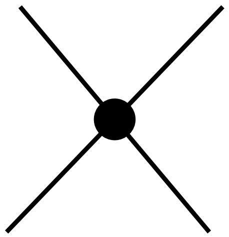
bent arm configuration
24 rotational orientations



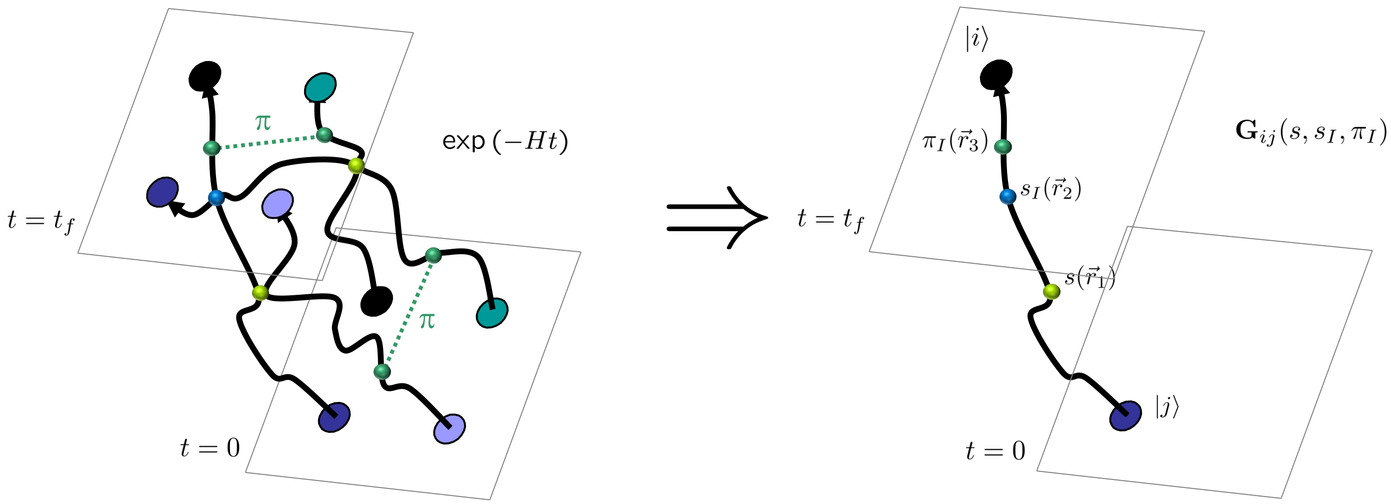
MONTE CARLO with AUXILIARY FILEDS

- Contact interactions represented by auxiliary fields s, s_I

$$\exp(\rho^2/2) \propto \int_{-\infty}^{+\infty} ds\, \exp(-s^2/2 - s\rho)\ , \quad \rho \sim N^\dagger N$$



- Correlation function = path-integral over pions & auxiliary fields

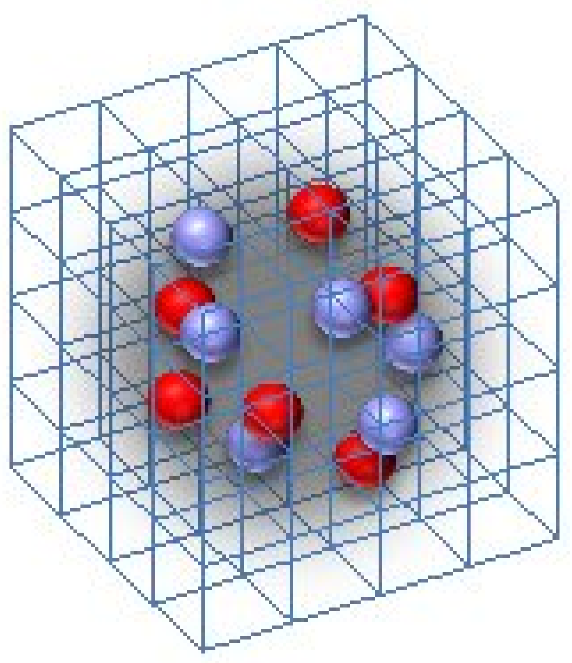


COMPUTATIONAL EQUIPMENT

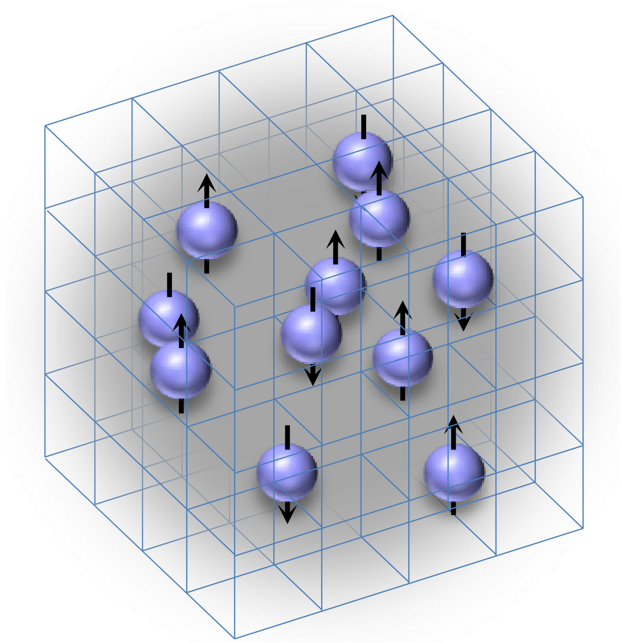
- Past = JUGENE (BlueGene/P)
- Present = JUQUEEN (BlueGene/Q)

Nuclear lattice simulations – Results –

nuclei



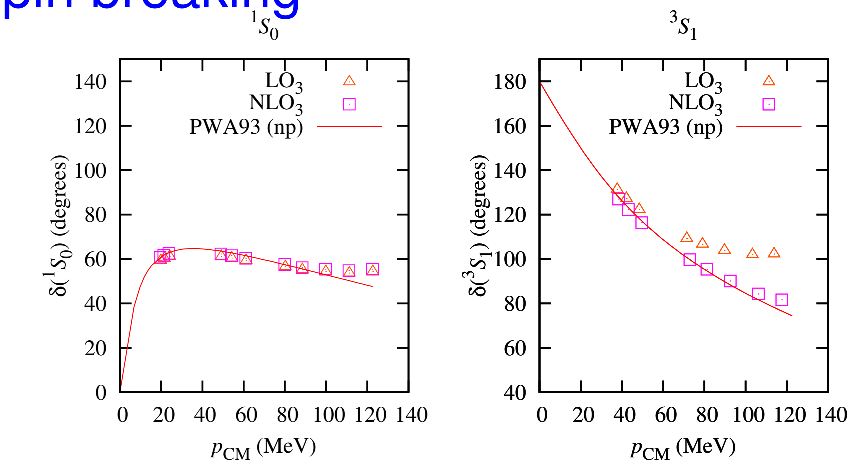
neutron matter



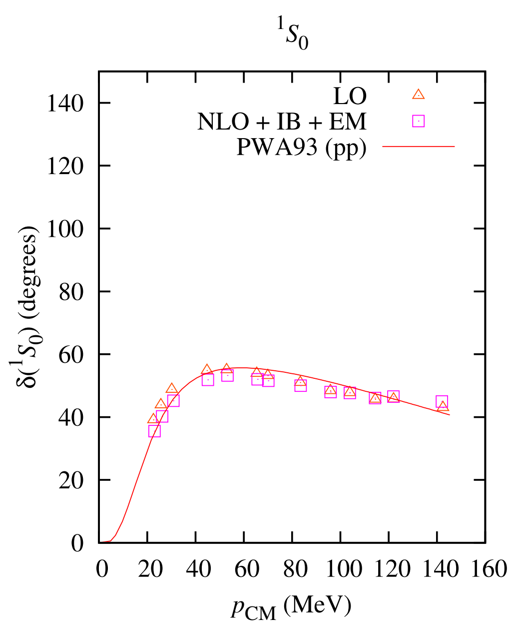
FIXING PARAMETERS & FIRST PREDICTIONS

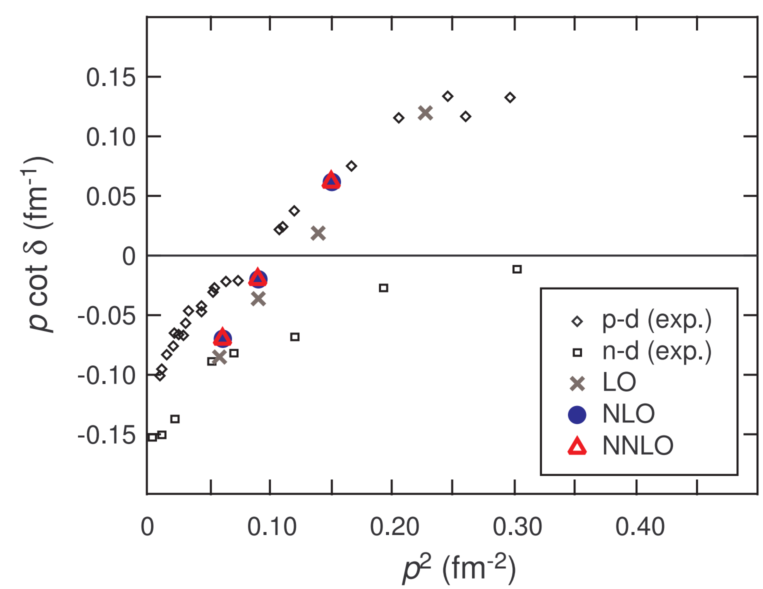
- work at NNLO including strong and em isospin breaking
- 9 NN LECs from np scattering and Q_d
- 2 LECs for isospin-breaking (np, pp, nn)
- 2 LECs D, E related to the leading 3NF

$\Rightarrow$ make predictions



- pp vs np scattering
- nd spin-3/2
  quartet channel
- . . .





Ground states

Epelbaum, Krebs, Lähde, Lee, UGM, arxiv:1208.1328

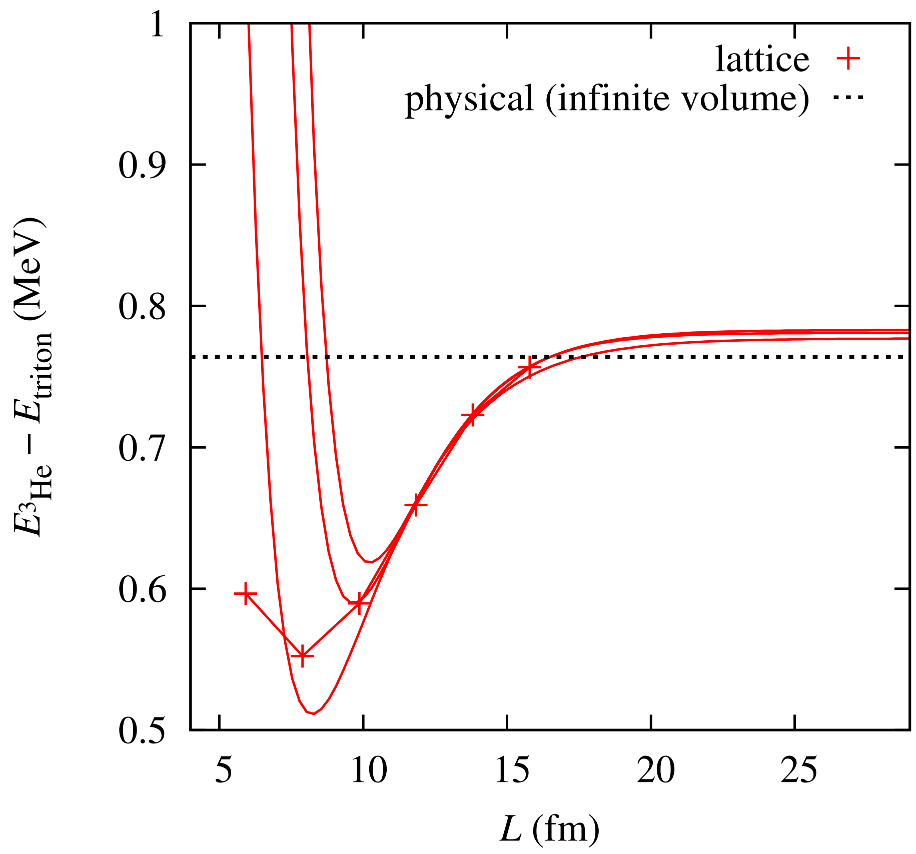
PREDICTIONS: TRITON & HELIUM-3

Epelbaum, Krebs, Lee, UGM, Phys. Rev. Lett. **104** (2010) 142501; Eur. Phys. J. **A 45** (2010) 335

- binding energies of 3N systems: $E(L) = \mathrm{B.E.} - \frac{a}{L}\exp(-bL)$

see also Hammer, Kreuzer (2011)

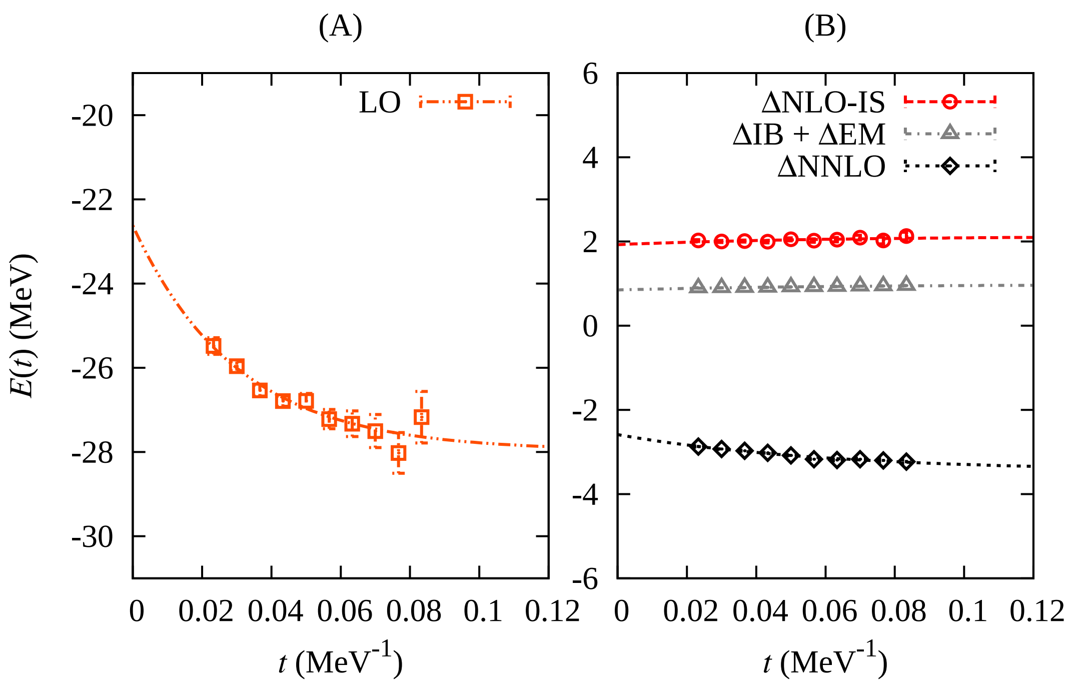
$\Rightarrow$ predict the energy difference $E(^3He) - E(^3H)$



0.78(5) MeV
0.76 MeV [exp.]

Ground state of ^{4}He

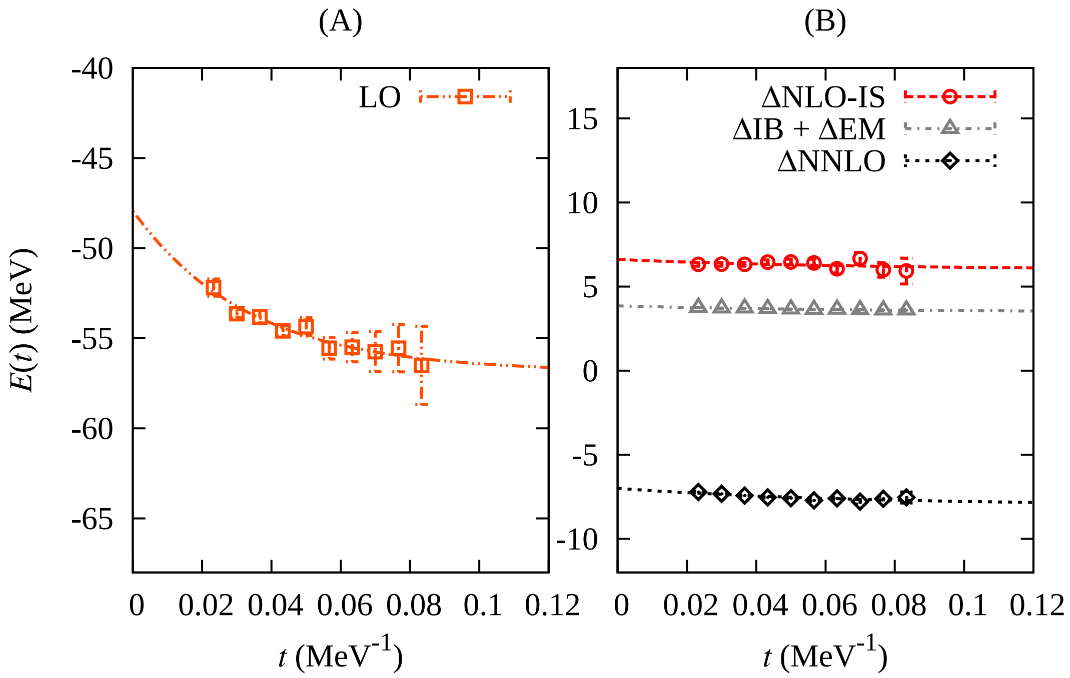
$$L = 11.8 \text{ fm}$$



LO ($\mathcal{O}(Q^0)$)	$-28.0(3)$ MeV
NLO ($\mathcal{O}(Q^2)$)	$-24.9(5)$ MeV
NNLO ($\mathcal{O}(Q^3)$)	$-28.3(6)$ MeV
Exp.	-28.3 MeV

Ground state of ^{8}Be

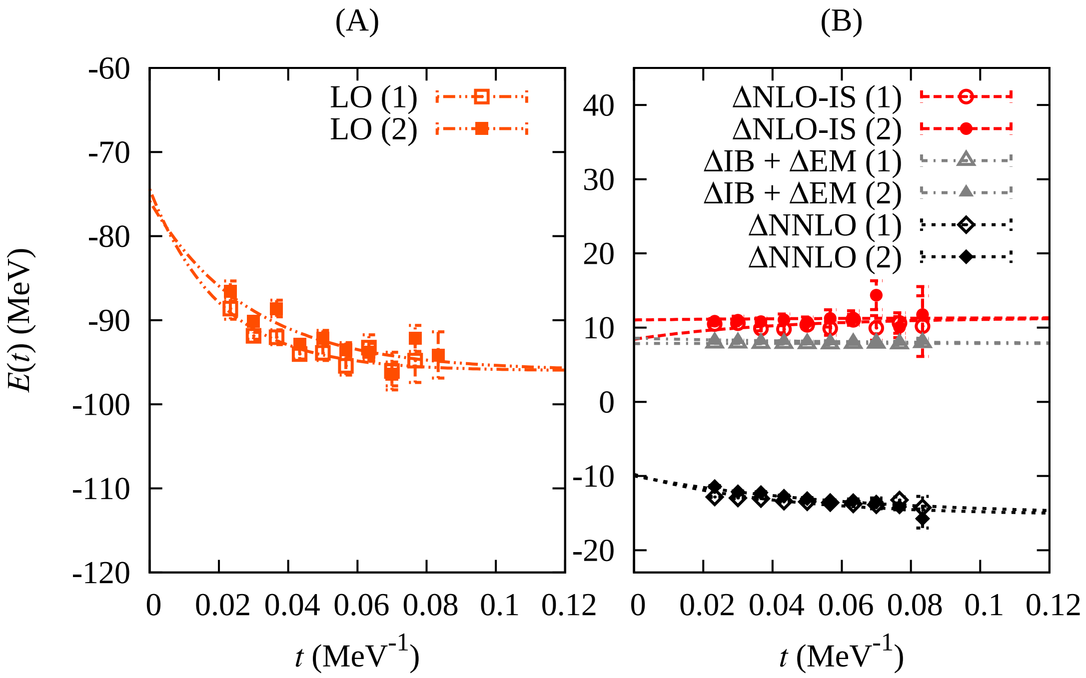
$$L = 11.8 \text{ fm}$$



LO ($\mathcal{O}(Q^0)$)	$-57(2)$ MeV
NLO ($\mathcal{O}(Q^2)$)	$-47(2)$ MeV
NNLO ($\mathcal{O}(Q^3)$)	$-55(2)$ MeV
Exp.	-56.5 MeV

Ground state of ^{12}C

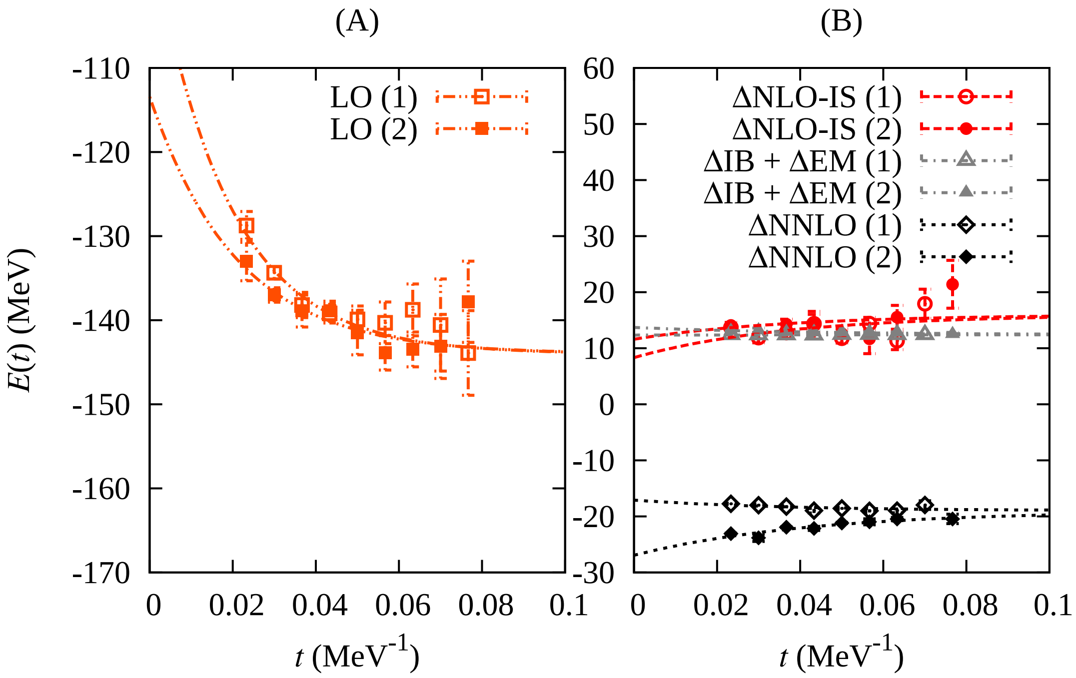
$$L = 11.8\ \mathrm{fm}$$



LO ($\mathcal{O}(Q^0)$)	$-96(2)$ MeV
NLO ($\mathcal{O}(Q^2)$)	$-77(3)$ MeV
NNLO ($\mathcal{O}(Q^3)$)	$-92(3)$ MeV
Exp.	-92.2 MeV

Ground state of ^{16}O

$$L = 11.8 \text{ fm}$$

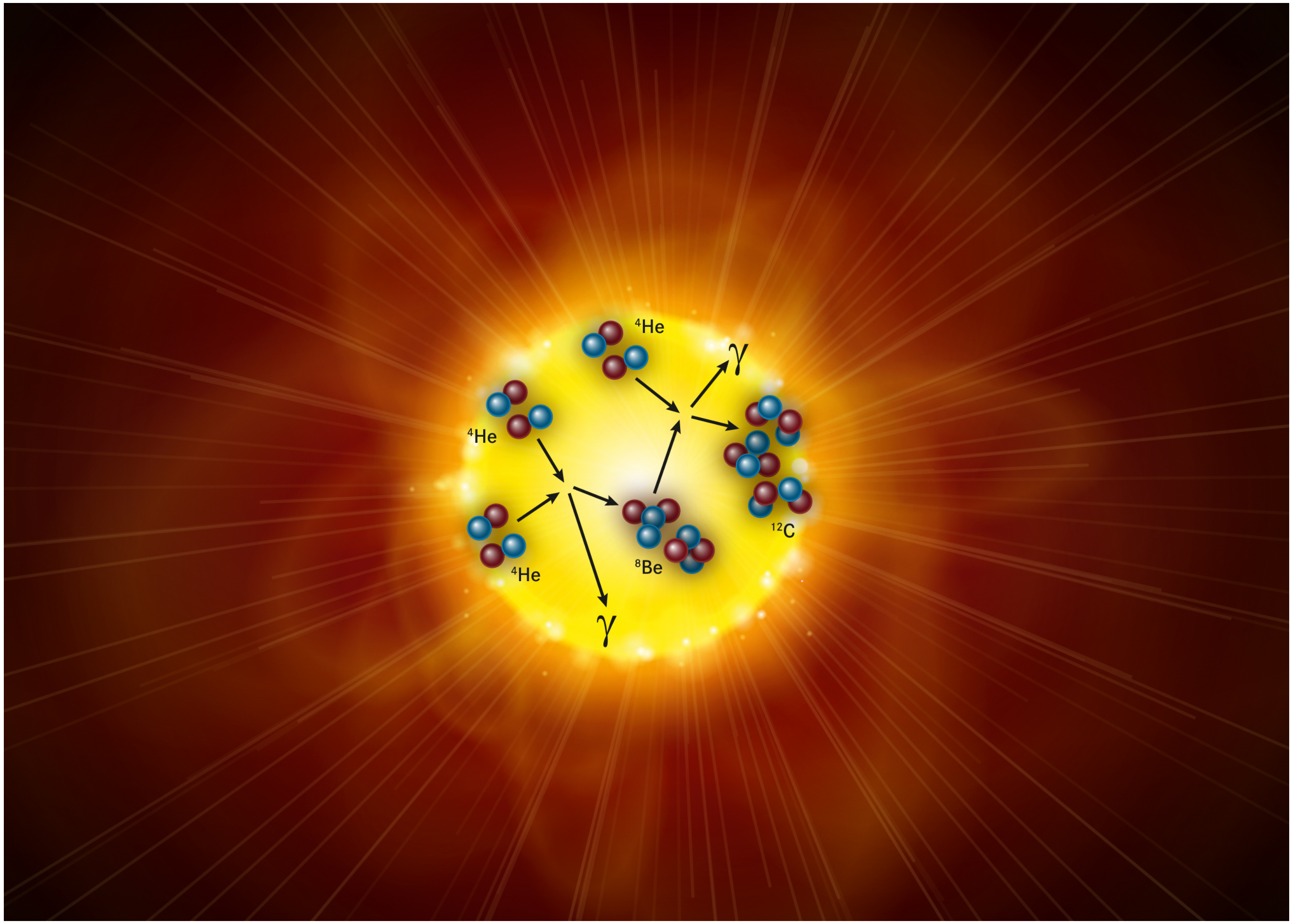


to be published

LO ($\mathcal{O}(Q^0)$)	$-144(4)$ MeV
NLO ($\mathcal{O}(Q^2)$)	$-116(6)$ MeV
NNLO ($\mathcal{O}(Q^3)$)	$-135(6)$ MeV
Exp.	-127.6 MeV

SPECTRUM OF ^{12}C & the HOYLE STATE

Epelbaum, Krebs, Lee, UGM, Phys. Rev. Lett. **106** (2011) 192501
Viewpoint: Hjorth-Jensen, Physics **4** (2011) 38
Epelbaum, Krebs, Lähde, Lee, UGM, arxiv:1208.1328 (numbers from this ref.)

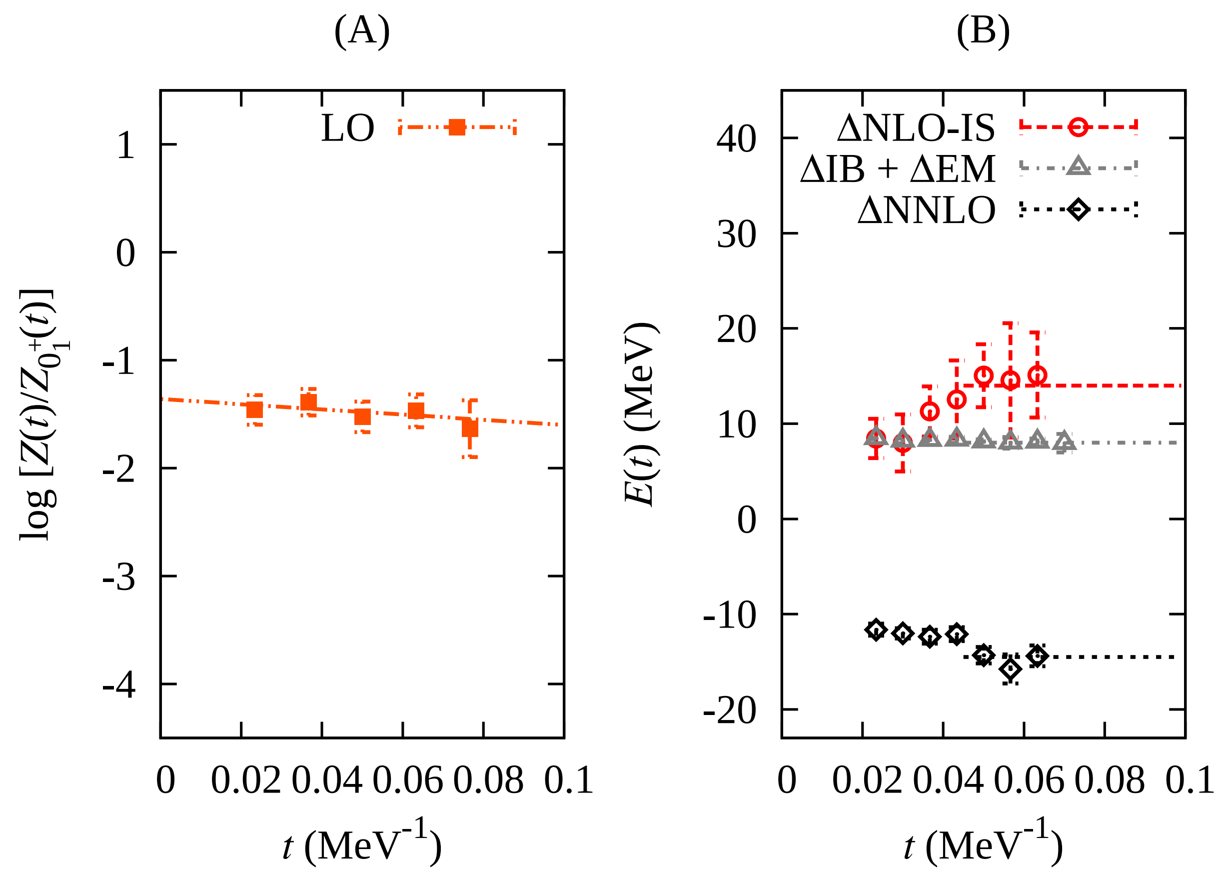


EXCITED STATES of ^{12}C

- Lowest excited state is $\mathbf{2_1^+}$ (as in nature)

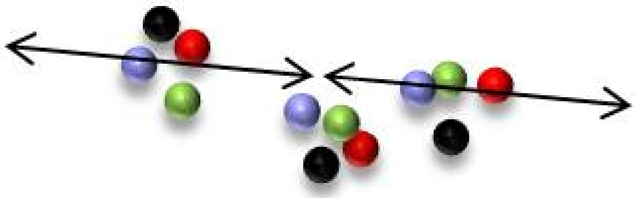
$E(2_1^+) = -89(3)$ MeV

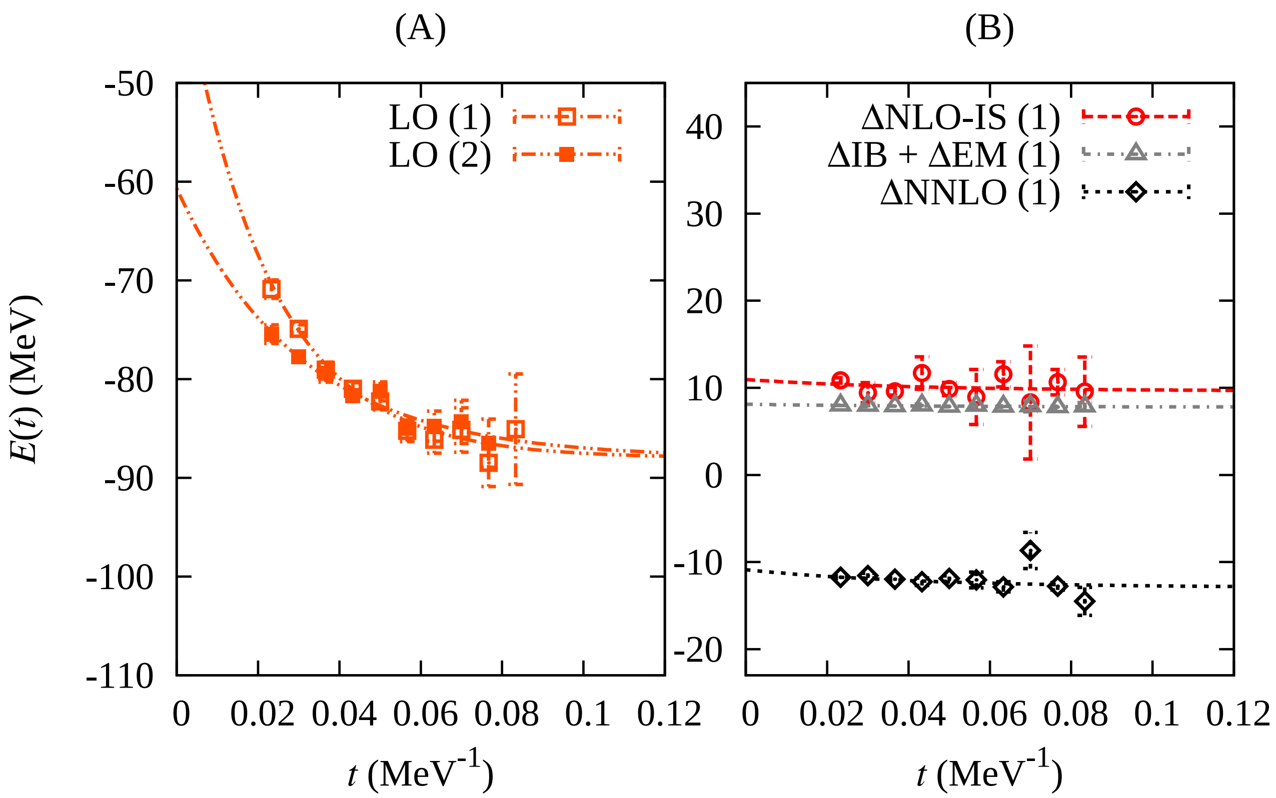
$[-87.7$ MeV]



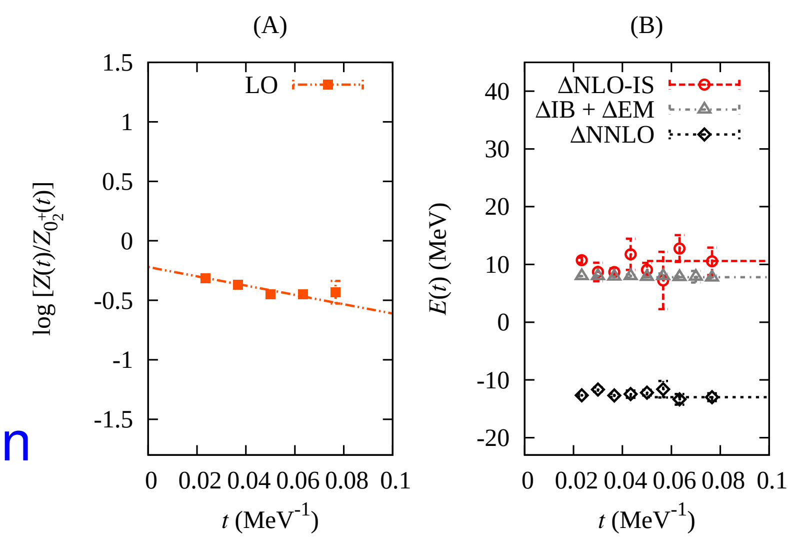
THE HOYLE STATE (0_2^+)

- energy: $E(0_2^+) = -85(3)$ MeV
- close to $E(^4\mathrm{He}) + E(^8\mathrm{Be}) = -83.3(2.0)$ MeV
- structure: “bent” alpha-chain like (not “BEC”)





A HOYLE STATE EXCITATION (2_2^+)



- a 2^+ state 2 MeV above the Hoyle state
- interpretation:
  a rotational band of the Hoyle state
  generated from excitations of the alpha-chain
- what's in the data ?
  a 2^+ state 3.51 MeV above the Hoyle state seen in $^{11}B(d,n)^{12}C$
  not included in the level scheme! Ajzenberg-Selove, Nucl. Phys. A506 (1990) 1

  a 2^+ state 3.8(4) MeV above the Hoyle state seen in $^{12}C(\alpha,\alpha)^{12}C$
  Bency John et al., Phys. Rev. C 68 (2003) 014305
- and much more, see next slide

$\Rightarrow$ ab initio prediction requires experimental confirmation

SPECTRUM OF ^{12}C

- Summarizing the results for carbon-12:

	0_1^+	2_1^+	0_2^+	2_2^+
LO	$-96(2)$ MeV	$-94(2)$ MeV	$-89(2)$ MeV	$-88(2)$ MeV
NLO	$-77(3)$ MeV	$-74(3)$ MeV	$-72(3)$ MeV	$-70(3)$ MeV
NNLO	$-92(3)$ MeV	$-89(3)$ MeV	$-85(3)$ MeV	$-83(3)$ MeV
Exp.	-92.16 MeV	-87.72 MeV	-84.51 MeV	$-82.6(1)$ MeV [1,2] $-82.32(6)$ MeV [3] $-81.1(3)$ MeV [4] $-82.13(11)$ MeV [5]

[1] Freer et al., Phys. Rev. C 80 (2009) 041303
[2] Zimmermann et al., Phys. Rev. C 84 (2011) 027304
[3] Hyldegaard et al., Phys. Rev. C 81 (2010) 024303
[4] Itoh et al., Phys. Rev. C 84 (2011) 054308
[5] Weller et al., in preparation

- importance of consistent 2N & 3N forces
- good agreement w/ experiment, can be improved
- test of the *Anthropic Principle* possible, intriguing results

A FIRST LOOK at EM PROPERTIES

- LO em results obtained (NLO/NNLO requires longer time simulations)

Observable	LO theory	Experiment
r($\mathbf{0_1^+}$) [fm]	2.2(2)	2.47(2)
Q($\mathbf{2_1^+}$) [e$^{\mathbf{2}}$ fm]	6(2)	6(3)
B(E2;$\mathbf{2_1^+} \to \mathbf{0_1^+}$) [e$^{\mathbf{2}}$ fm^{4}]	5(2)	7.6(4)
B(E2;$\mathbf{2_1^+} \to \mathbf{0_2^+}$) [e$^{\mathbf{2}}$ fm^{4}]	1.5(7)	2.6(4)
m(E0;$\mathbf{0_2^+} \to \mathbf{0_1^+}$) [e fm$^{\mathbf{2}}$]	3(1)	5.5(1)

- satisfying agreement for LO calculation
- other radii, quadrupole moments and transition strengths also computed
- FMD and cluster models predict e.g. $r(0_2^+) = (3.5 - 3.9)$ fm
- higher order corrections are in the works

STATUS SUMMARY

2012/2

- detailed investigation of the ^{12}C nucleus, in part. the structure of the Hoyle state

→ achieved, publication submitted, more details to come

2013

- detailed investigation (spectrum and wave function) of ^{14}N

→ postponed, do the more interesting nucleus ^{16}O first (same CPU time)

2014

- detailed investigation of spectrum and wave function of ^{16}O

→ ground state done, spectrum runs start 09/2012 (JUQUEEN and RWTH Bull Cl.)

2015

- formulation of the lattice set-up to perform simulations of light hyper-nuclei based on the leading order hyperon-nucleon interactions

→ start to develop a new method w/ hyperons in nuclear background field

2016/1

- lattice simulations of light hyper-nuclei

→ if method works, will be done . . .

Testing the Anthropic Principle

MC ANALYSIS of the AP

- consider QCD only $\rightarrow$ calculate $\partial\Delta E/\partial M_\pi$

- relevant quantities (energy *differences*)

$$\boxed{\Delta E_h \equiv E^*_{12} - E_8 - E_4, \quad \Delta E_b \equiv E_8 - 2E_4} \qquad \Delta E_c \equiv E^*_{12} - E_{12}$$

- energy differences depend on parameters of QCD (LO analysis)

$$\boxed{E_i = E_i\left(M_\pi^{\rm OPE}, m_N(M_\pi), \tilde{g}_{\pi N}(M_\pi), C_0(M_\pi), C_I(M_\pi)\right)}$$

$$\tilde{g}_{\pi N} \equiv \frac{g_A}{2F_\pi}$$

- remember: $M^2_{\pi^\pm} \sim (m_u + m_d)$

$\Rightarrow$ quark mass dependence $\equiv$ pion mass dependence

PION MASS VARIATIONS

- consider pion mass changes as *small perturbations*

$$
\left.\frac{\partial E_i}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} = \left.\frac{\partial E_i}{\partial M_\pi^{\rm OPE}}\right|_{M_\pi^{\rm phys}} + x_1 \left.\frac{\partial E_i}{\partial m_N}\right|_{m_N^{\rm phys}} + x_2 \left.\frac{\partial E_i}{\partial \tilde{g}_{\pi N}}\right|_{\tilde{g}_{\pi N}^{\rm phys}}
+ x_3 \left.\frac{\partial E_i}{\partial C_0}\right|_{C_0^{\rm phys}} + x_4 \left.\frac{\partial E_i}{\partial C_I}\right|_{C_I^{\rm phys}}
$$

with

$$
x_1 \equiv \left.\frac{\partial m_N}{\partial M_\pi}\right|_{M_\pi^{\rm phys}}, \; x_2 \equiv \left.\frac{\partial \tilde{g}_{\pi N}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}}, \; x_3 \equiv \left.\frac{\partial C_0}{\partial M_\pi}\right|_{M_\pi^{\rm phys}}, \; x_4 \equiv \left.\frac{\partial C_I}{\partial M_\pi}\right|_{M_\pi^{\rm phys}}
$$

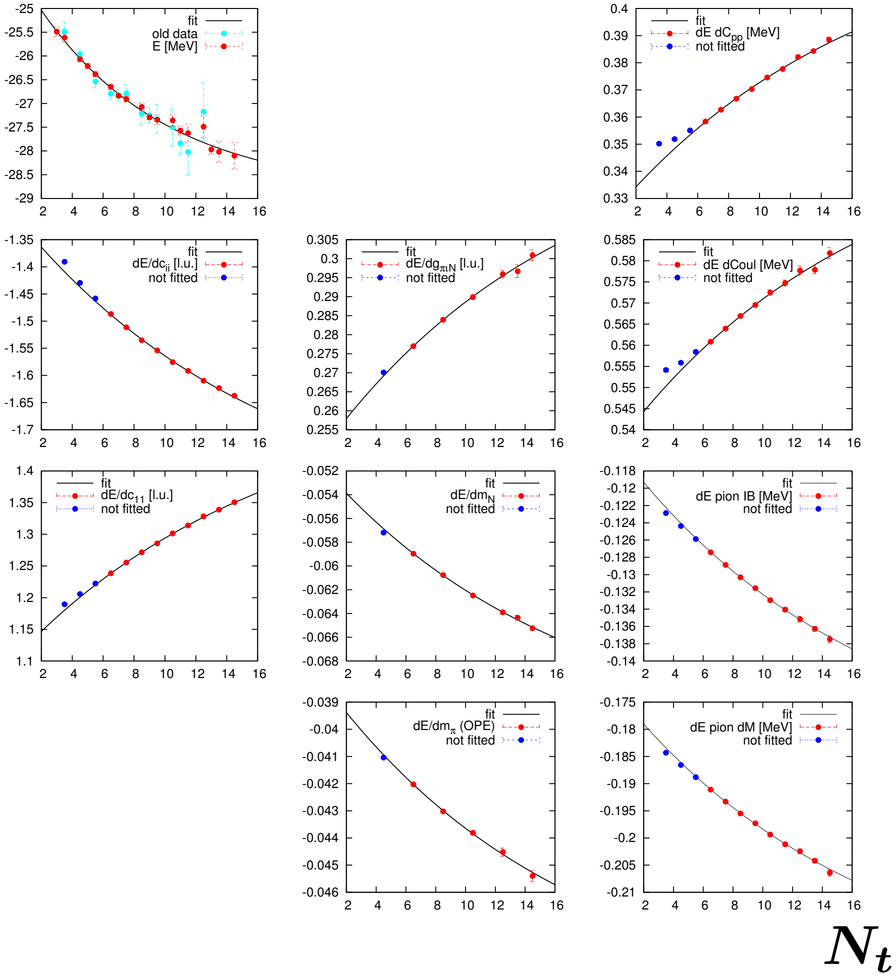
$\Rightarrow$ problem reduces to the calculation of the various derivatives using AFQMC and the determination of the x_i

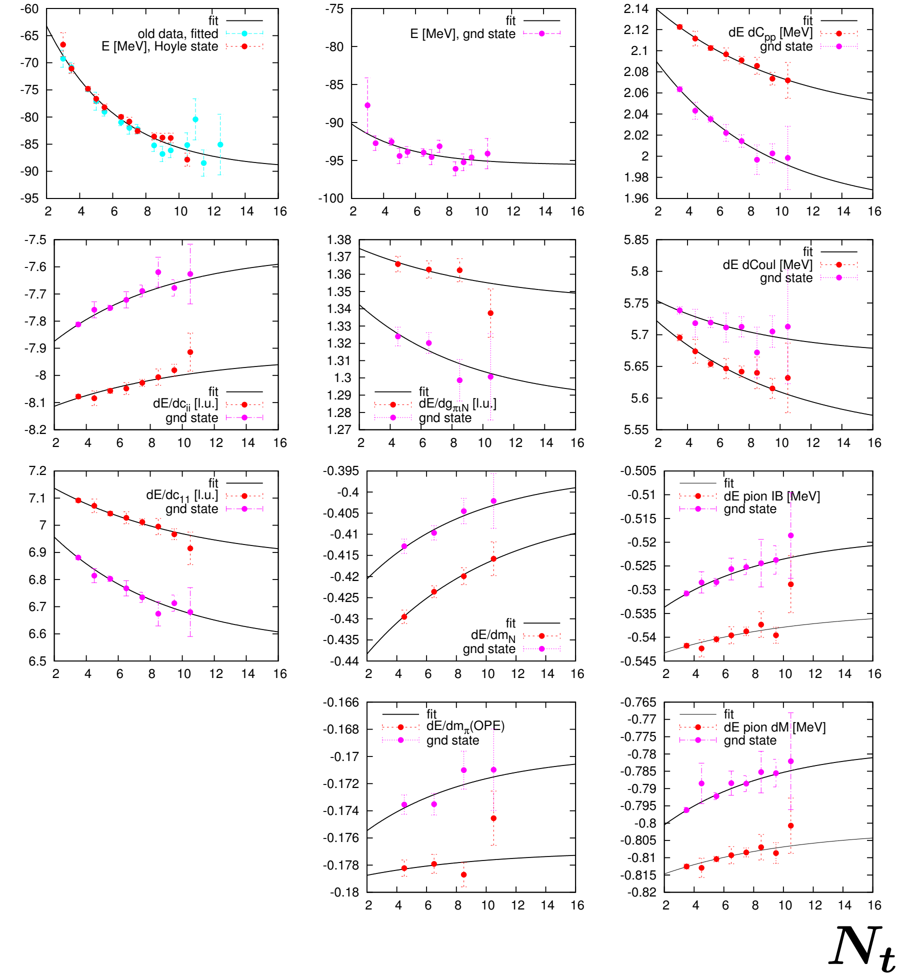
- x_1 and x_2 can be obtained from LQCD plus CHPT
- x_3 and x_4 can be obtained from two-body scattering and its M_π-dependence

AFQMC RESUTS for the DERIVATIVES

- ^{4}He
- ^{12}C(0_2^+)

$$E(N_t) = E(\infty) + \text{const}\,\exp(-N_t/\tau)$$





DETERMINATION of the x_i

- x_1 from the quark mass expansion of the nucleon mass: $x_1 \simeq 0.8 \pm 0.2$
- x_2 from the quark mass expansion of the pion decay constant and the nucleon axial-vector constant: $x_2 \simeq -0.056 \ldots 0.008$
- x_3 and x_4 can be obtained from a two-nucleon scattering analysis & can be deduced from:

$$-\frac{\partial a^{-1}}{\partial M_\pi} \equiv \frac{A}{aM_\pi} = \frac{1}{\pi L} S'(\eta) \frac{\partial \eta}{\partial M_\pi} \, , \quad \eta \equiv m_N E \left(\frac{L}{2\pi}\right)^2$$

$\Rightarrow$ while this can straightforwardly be computed, we prefer to use a representation that substitutes x_3 and x_4 by:

$$\left.\frac{\partial a_s^{-1}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} \, , \quad \left.\frac{\partial a_t^{-1}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}}$$

$\Rightarrow$ we are ready to study the pertinent energy differences

RESULTS

- putting pieces together:

$$\left.\frac{\partial \Delta E_h}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} = -0.455(35)\left.\frac{\partial a_s^{-1}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} - 0.744(24)\left.\frac{\partial a_t^{-1}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} + 0.056(10)$$

$$\left.\frac{\partial \Delta E_b}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} = -0.117(34)\left.\frac{\partial a_s^{-1}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} - 0.189(24)\left.\frac{\partial a_t^{-1}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} + 0.012(9)$$

$$\left.\frac{\partial \Delta E_c}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} = -0.07(3)\left.\frac{\partial a_s^{-1}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} - 0.14(2)\left.\frac{\partial a_t^{-1}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} + 0.017(9)$$

- x_1 and x_2 only affect the small constant terms
- also calculated the shifts of the individual energies (not shown here)

INTERPRETATION

- $(\partial \Delta E_h/\partial M_\pi)/(\partial \Delta E_b/\partial M_\pi) \simeq 4$

  $\Rightarrow$ ΔE_h and ΔE_b cannot be independently fine-tuned

- Within error bars, $\partial \Delta E_h/\partial M_\pi$ & $\partial \Delta E_b/\partial M_\pi$ appear unaffected by the choice of x_1 and x_2 $\rightarrow$ indication for α-clustering

- For ΔE_h & ΔE_b, the dependence on M_π is small when

$$\boxed{\partial a_s^{-1}/\partial M_\pi \simeq -1.6 \times \partial a_t^{-1}/\partial M_\pi}$$

- the triple alpha process is controlled by :

$$\Delta E_{h+b} \equiv \Delta E_h + \Delta E_b = E_{12}^\star - 3E_4$$

$$\boxed{\left.\frac{\partial \Delta E_{h+b}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} = -0.571(14)\left.\frac{\partial a_s^{-1}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} - 0.934(11)\left.\frac{\partial a_t^{-1}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} + 0.069(6)}$$

$\Rightarrow$ so what can we say about the quark mass dependence of the scattering lengths?

CONSTRAINTS on the SCATTERING LENGTHS

- Quark mass dependence of hadron properties: $\dfrac{\delta O_H}{\delta m_f} \equiv K_H^f \dfrac{O_H}{m_f}\,, \quad f = u, d, s$

- NN scattering lengths as a function of M_π: $-\dfrac{\partial a_{s,t}^{-1}}{\partial M_\pi} \equiv \dfrac{A_{s,t}}{a_{s,t} M_\pi}, \quad A_{s,t} \equiv \dfrac{K^q_{a_{s,t}}}{K^q_\pi}$

- earlier determinations from chiral EFT at NLO

Beane, Savage (2003), Epelbaum, Glöckle, UGM (2003)

- new determination at NNLO:

Epelbaum et al. (2012)

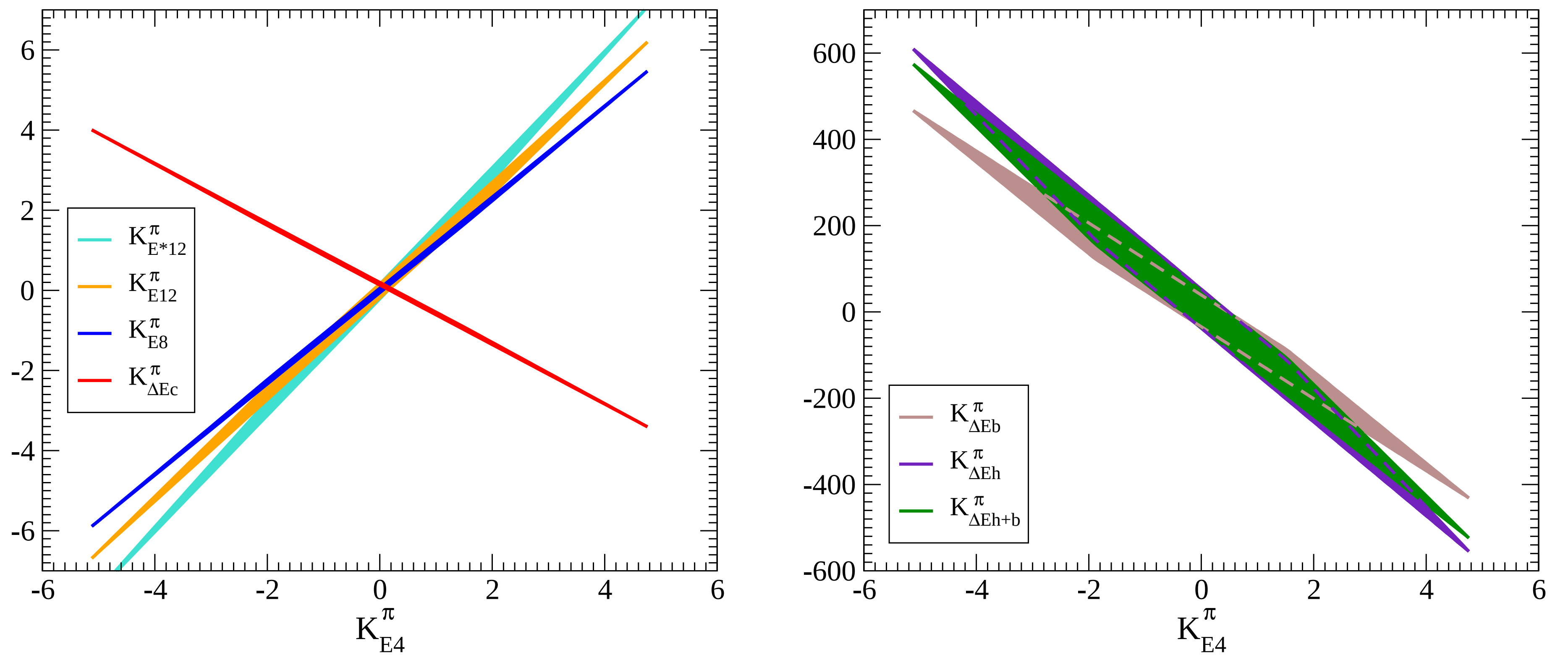
$$K^q_{a_s} = 2.3^{+1.9}_{-1.8}\,,\ K^q_{a_t} = 0.32^{+0.17}_{-0.18} \to \frac{\partial a_t^{-1}}{\partial M_\pi} = -0.18^{+0.10}_{-0.10}\,,\ \frac{\partial a_s^{-1}}{\partial M_\pi} = 0.29^{+0.25}_{-0.23}$$

- note the *magical* central value:

$$\boxed{\frac{\partial a_s^{-1}/\partial M_\pi}{\partial a_t^{-1}/\partial M_\pi} \simeq -1.6^{+1.0}_{-1.7}}$$

CORRELATIONS

- vary the quark mass derivatives of $a_{s,t}^{-1}$ within $-1, \ldots, +1$:



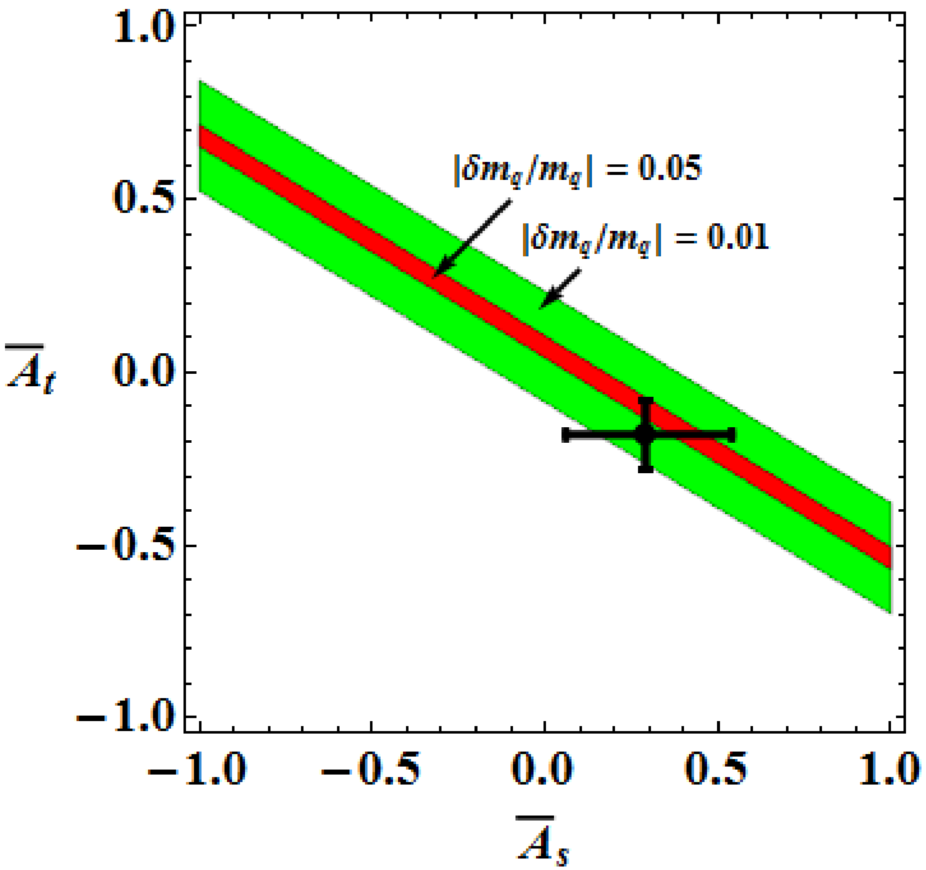
- clear correlations: α-particle BE and the energies/energy differences

$\Rightarrow$ anthropic or non-anthropic scenario depends on whether the ^{4}He BE moves!

THE END-OF-THE-WORLD PLOT

- $|\delta(\Delta E_{h+b})| < 100$ keV

$$\rightarrow \boxed{\left|\left(0.571(14)\,\bar{A}_s + 0.934(11)\,\bar{A}_t - 0.069(6)\right)\frac{\delta m_q}{m_q}\right| < 0.0015}$$

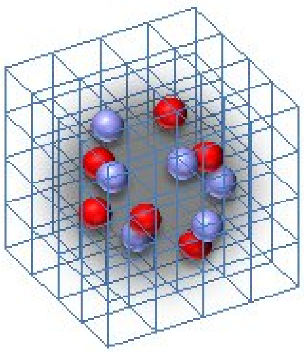


$$\bar{A}_{s,t} \equiv \left.\frac{\partial a_{s,t}^{-1}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}}$$

SUMMARY & OUTLOOK

- Nuclear lattice simulations as a new quantum many-body approach
- Formulate continuum EFT on space-time lattice $V = L_s \times L_s \times L_s \times L_t$
- New method to extract phase shifts & mixing angles
- Fix parameters in few-nucleon systems $\rightarrow$ predictions
- Promising results for $A = 2, 3, 4, 8, 12, 16$ at NNLO
- ^{12}C spectrum at NNLO $\rightarrow$ Hoyle state & 2^+ excitation
- First ever ab initio MC calculation of ^{16}O
- Testing the anthropic principle $\rightarrow$ strong correlations of α-cluster type

  $\Rightarrow$ the Hoyle state does not appear anthropic (Coulomb to be done)



$\Rightarrow$ larger A and higher precision