Light Hypernuclei

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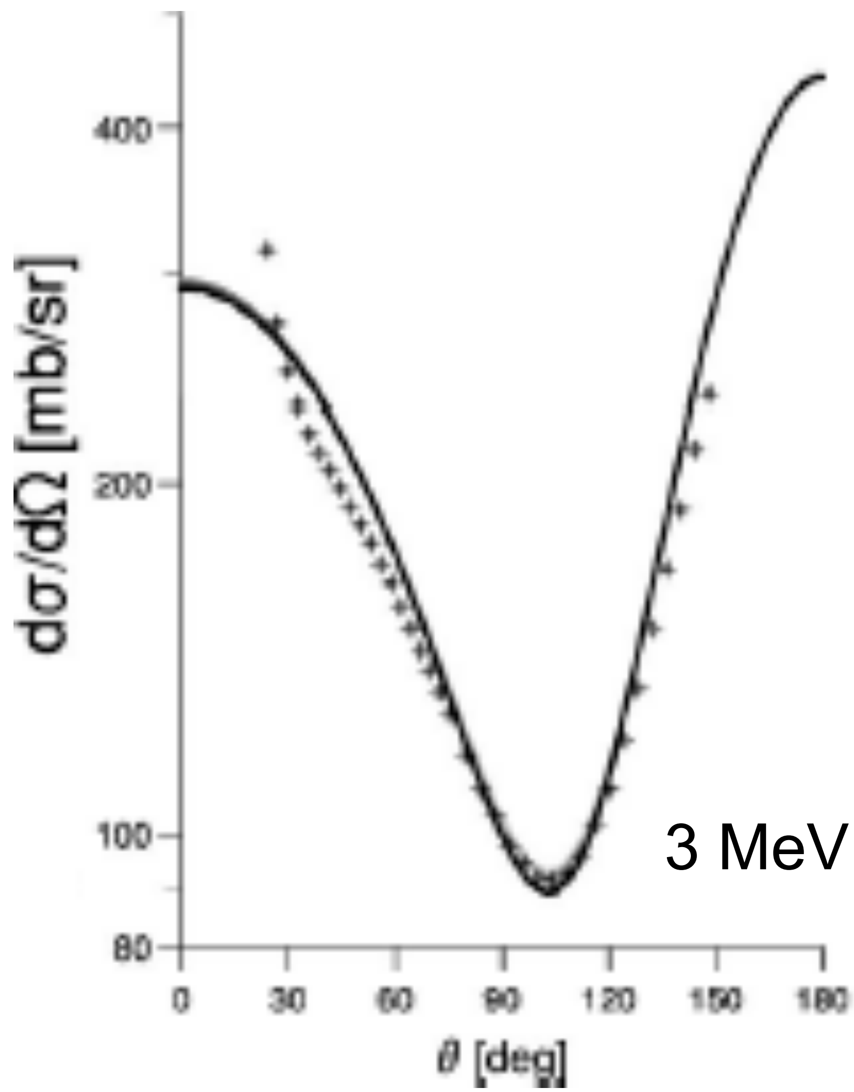
- few-nucleon systems & NN forces
- realistic, phenomenological YN interactions & light hypernuclei
  - numerical technique
  - predictions based on realistic models
- chiral effective theory & light hypernuclei
  - separation energies based on chiral interactions
  - CSB of four-body hypernuclei
- conclusions & outlook

Few-nucleon systems

Several NN force models (AV18, CD-Bonn, Nijmegen, ...) describe the data **(~ 4000)** up to the pion production threshold perfectly using **~ 40** parameters

Long-range part is driven by **one-pion exchange**

Predictions based on NN forces are reasonable:
Many low energy few-nucleon observables are well & model independently described !



(see e.g. Witała et al., 2001)

→ Approximation to the nuclear Hamiltonian does not seem to be too bad, but .....

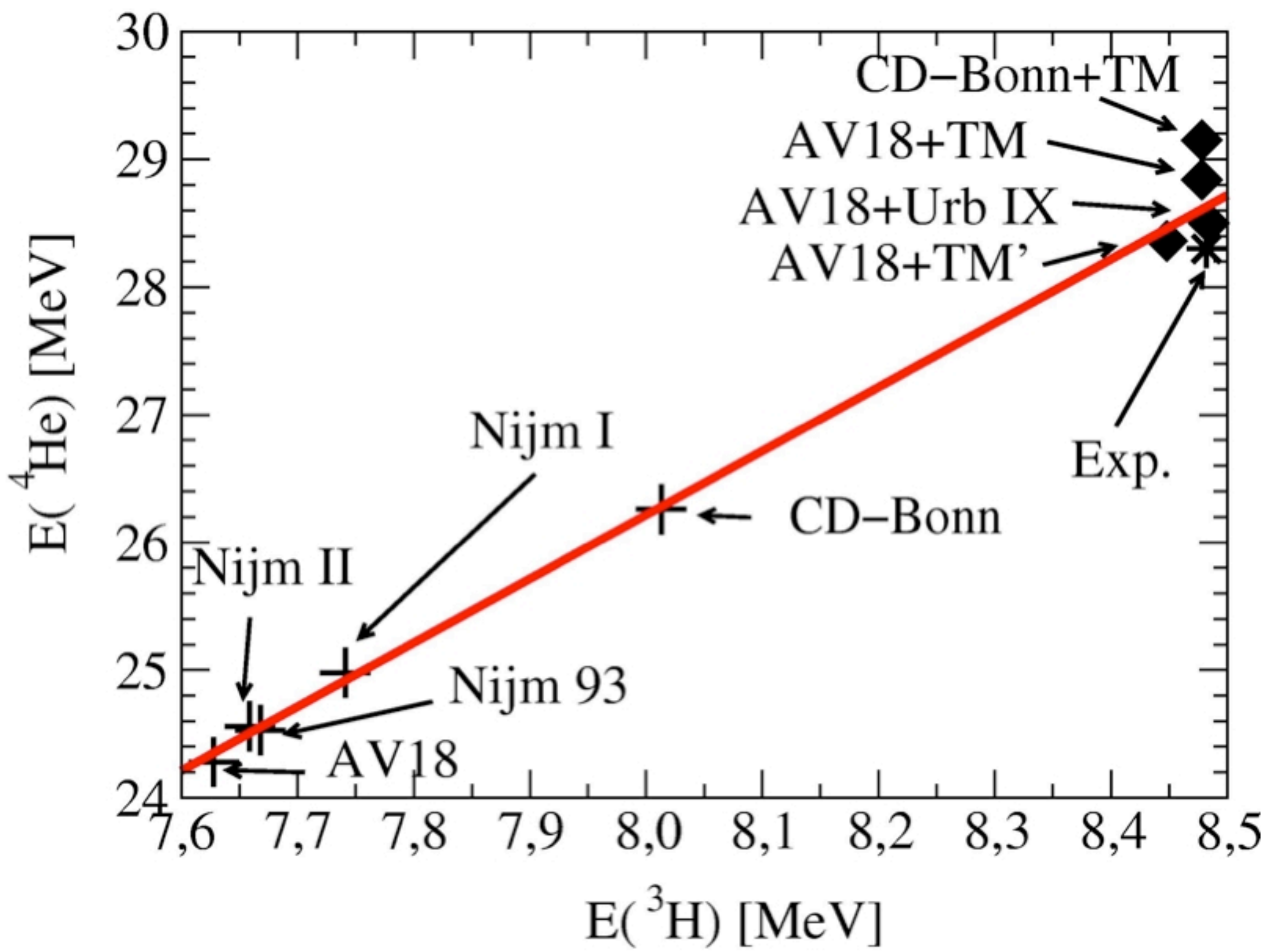
Few-nucleon bound states

Binding energies are not model-independent

& the results do not agree with experiment

	^3H	^4He
CD-Bonn	-8.013	-26.23
AV18	-7.628	-24.25
Nijm I	-7.741	-24.99
Nijm II	-7.659	-24.55
Expt	-8.482	-28.30

(see e.g. A.N. et al., 2002)



Even phase shift equivalent NN forces predict different binding energies!
3NF's are quantitatively important.

Cancelation of kinetic and potential energy!
Small parts of the nuclear Hamiltonian are relevant!

Do we have a chance to predict hypernuclear binding energies?

Hypernuclear interactions

Traditional „realistic“ interaction models for

YN interactions (Jülich 89/04, Nijmegen 89/97a-f, ...)

are based on OBE exchange and some flavor-SU(3) symmetry

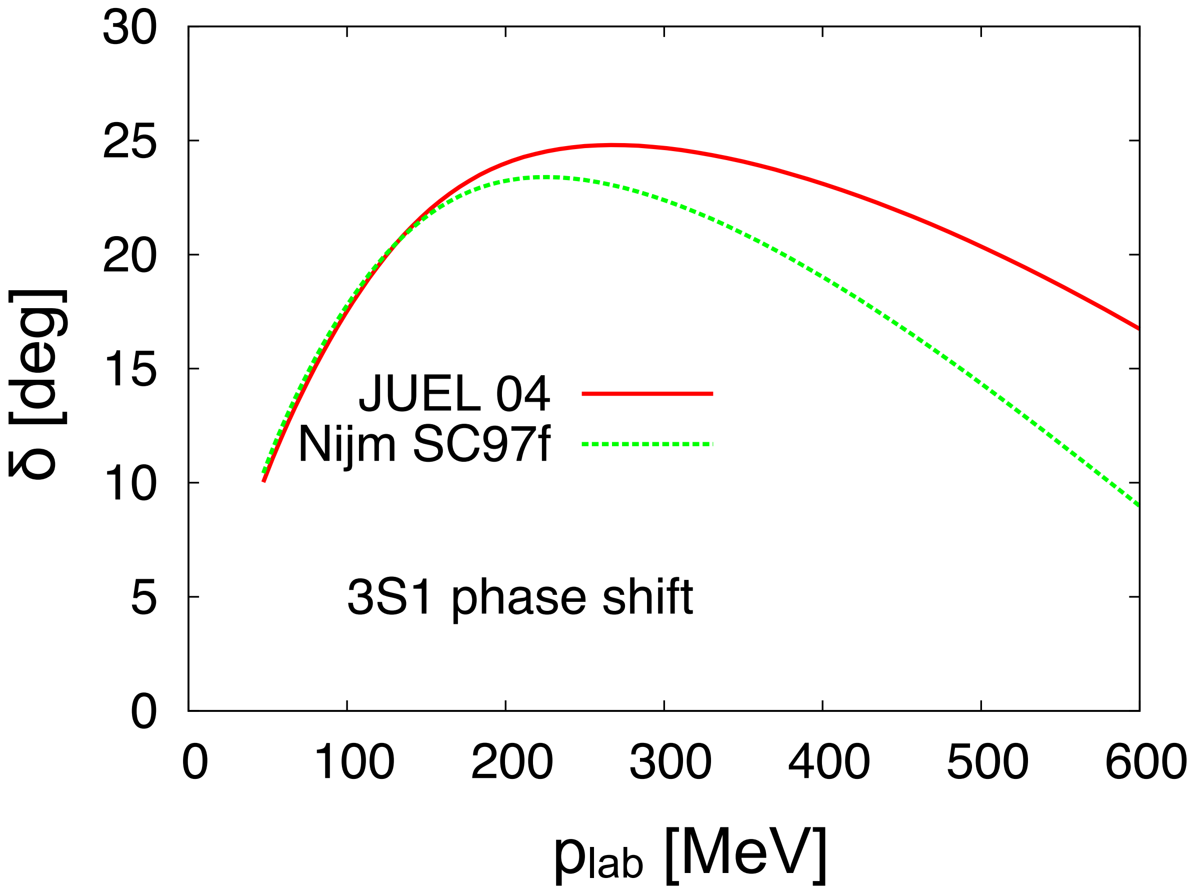
used in fit: **35** YN data, no YN bound state

→

all models describe the two-body data

but models are ***not*** equivalent (≠ NN case)

realistic models include Λ-Σ conversion



How to further constrain the YN interactions?

Hypernuclei

- *ΛN interaction generally weaker than the NN interaction*

  → *naively:* ***core nucleus + hyperons***

  Does this suppress 3BF's?

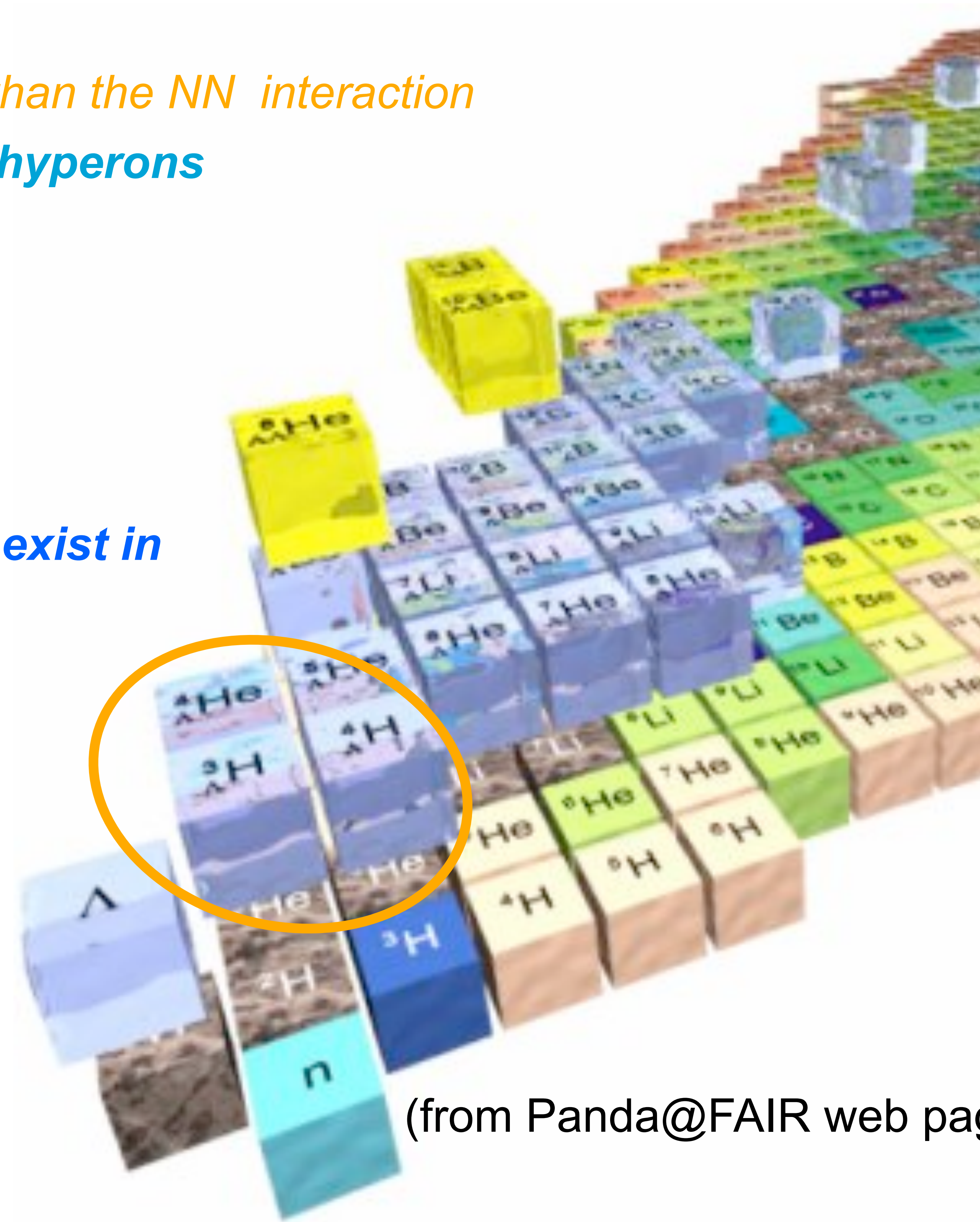
- *no Pauli blocking of Λ in nuclei*

  → *nuclear structure,*
  even light hypernuclei exist in several spin states

- ***non-trivial constraints***

  on the YN interaction even from lightest ones

  → *see results in the following*



(from Panda@FAIR web page)

Light hypernuclei

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no ΛN bound state

${}^{3}_{\Lambda}\mathrm{H}\ (pn\Lambda)$	$T=0$		$E_\Lambda = 0.13 \pm 0.05\ \mathrm{MeV}$	
${}^{4}_{\Lambda}\mathrm{H} \Big/ {}^{4}_{\Lambda}\mathrm{He}$	$T=\frac{1}{2}$	$J^\pi = 0^+$	$E_\Lambda = 2.04 \pm 0.04\ \mathrm{MeV}$	$E_\Lambda = 2.39 \pm 0.03\ \mathrm{MeV}$
		$J^\pi = 1^+$	$E_\Lambda = 1.00 \pm 0.06\ \mathrm{MeV}$	$E_\Lambda = 1.24 \pm 0.05\ \mathrm{MeV}$
~~${}^{5}_{\Lambda}\mathrm{He}$~~	~~$T=0$~~	~~$J^\pi = \frac{1^+}{2}$~~	~~$E_\Lambda = 3.12 \pm 0.02\ \mathrm{MeV}$~~	*not discussed here*

- quantum numbers as expected from core-Λ
- conventional to discuss Λ-separation energies
- unusually large CSB
- ${}^{5}_{\Lambda}\mathrm{He}$ has an unusually small separation energy (see e.g. Nemura et al., 2002)

Numerical technique

non-rel. Schrödinger equation $\Psi = G_0 V \Psi$ $G_0 = \frac{1}{E - H_0}$

→ decomposition in five **Yakubovsky components**

$$\Psi = (1+P)(\psi_{1A} + \psi_{1B} + \psi_{2A} + \psi_{2B}) + (1 - P_{12})(1+P)\psi_{1C}$$

solution of the **Yakubovsky equations**

$$\psi_{1A} = G_0 t_{12} P (\psi_{1A} + \psi_{1B} + \psi_{2A}) + (1 + G_0 t_{12}) G_0 V_{123}^{(3)} \Psi$$

$$\psi_{1B} = G_0 t_{12} ((1 - P_{12})(1 - P_{23}) \psi_{1B} + P \psi_{2B})$$

$$\psi_{1C} = G_0 t_{14} (\psi_{1A} + \psi_{1B} + \psi_{2A} - P_{12} \psi_{1C} + P_{12} P_{23} \psi_{1C} + P_{13} P_{23} \psi_{2B})$$

$$\psi_{2A} = G_0 t_{12} ((P_{12} - 1) P_{13} \psi_{1C} + \psi_{2B})$$

$$\psi_{2B} = G_0 t_{34} (\psi_{1A} + \psi_{1B} + \psi_{2A})$$

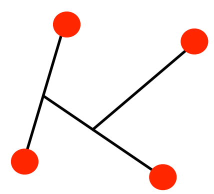
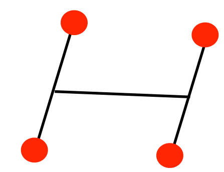
$(P = P_{12} P_{23} + P_{13} P_{23})$

→ **improved convergence in terms of partial waves**

Numerical technique

suitable basis system: Jacobi momentum states

- local and non-local interactions
- realistic interactions

$\langle$  $|\psi_{1A,B,C}\rangle$ $\langle$  $|\psi_{2A,B}\rangle$

- "natural" 3+1 or 2+2 coordinates for each Yakubovsky component
- several possibilities depending on the position of the hyperon

- 3 momenta discretized with 40-50 grid points
- thousands of partial waves contribute

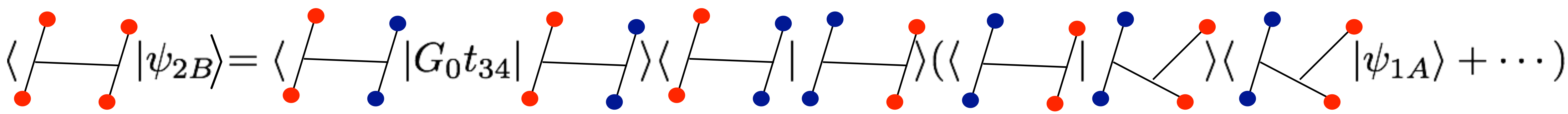
$$l_{12} + l_3 + l_4 \leq l_{sum}$$

l_{sum}	# of pw	E_Λ (1^+) [keV]
6	7893	877
8	15703	900
10	27266	903
12	42629	903

up to **10^9 x 10^9** dimensional eigenvalue problem

Numerical technique

Iterative solution for states with largest eigenvalues requires application of kernel to states



- steps with block diagonal form allow to apply matrix to trial state
- additional steps to perform coordinate transformations

- still computationally demanding
- feasible on 30-200 cores with 2 GB per core

Now look at results of YN interaction models: Nijm SC97, Jülich, ...

Calculations include the full interaction: **Λ-Σ conversion**, tensor interactions, ...

For light hypernuclei we are able to predict binding energies without uncontrolled approximation

Numerical uncertainties are no issue

Effective ΛN interactions

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Λ-Σ conversion is an essential part of YN interactions

unpredictable, **spin and charge dependent** effects may occur when Σ are not taken into account explicitly

E.g. use $t_{\Lambda N}$ in Yakubovsky equations (here for a chiral interaction)

	w/ Σ	w/o Σ
E_Λ (0^+) [MeV]	1.47	1.01
E_Λ (1^+) [MeV]	0.71	0.49

By construction, this t-matrix corresponds to an effective ΛN interaction that is exactly phase equivalent to the original interaction

→ results are **not useful to adjust YN interactions**

Results: independence of NN force

${}^{4}_{\Lambda}\mathrm{He}$	0^+		1^+		Δ
	E_B	E_Λ	E_B	E_Λ	
Bonn B	-8.92	1.66	-8.04	0.80	0.84
Nijm 93	-8.55	1.54	-7.69	0.72	0.79
Nijm 93 + TM	-9.32	1.56	-8.35	0.70	0.82

YN interaction: **SC97e**

- binding energies mostly depend on the NN interaction because of the core
- Λ separation energies are not strongly dependent on the NN interaction

→

YN interaction can be discussed independent of an NN force model

the dependence on the NN force is no issue either, we may discuss hypernuclei without special attention to the NN force

but: never use effective ΛN interaction if you are interested in the YN force

Results: model-dependence

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	${}^{3}_{\Lambda}\mathrm{H}$ in MeV	${}^{4}_{\Lambda}\mathrm{He}$ 0^+ in MeV	${}^{4}_{\Lambda}\mathrm{He}$ 1^+ in MeV	${}^{1}a_{\Lambda p}$ in fm	${}^{3}a_{\Lambda p}$ in fm	P_{Σ}
SC97d	-	1.3	0.8	-1.7	-1.9	1.5 %
SC97e	0.02	1.5	0.7	-2.1	-1.8	1.6 %
SC97f	0.08	1.7	0.5	-2.5	-1.7	1.8 %
SC89	0.15	2.1	0.02	-2.6	-1.4	4.1 %
Jülich 04	0.13	1.9	2.3	-2.6	-1.7	0.9 %
Expt	0.13	2.4	1.2	?	?	-

- **none** of these interaction models predicts the hypernuclei correctly
- no strict relation of the scattering lengths to any separation energy

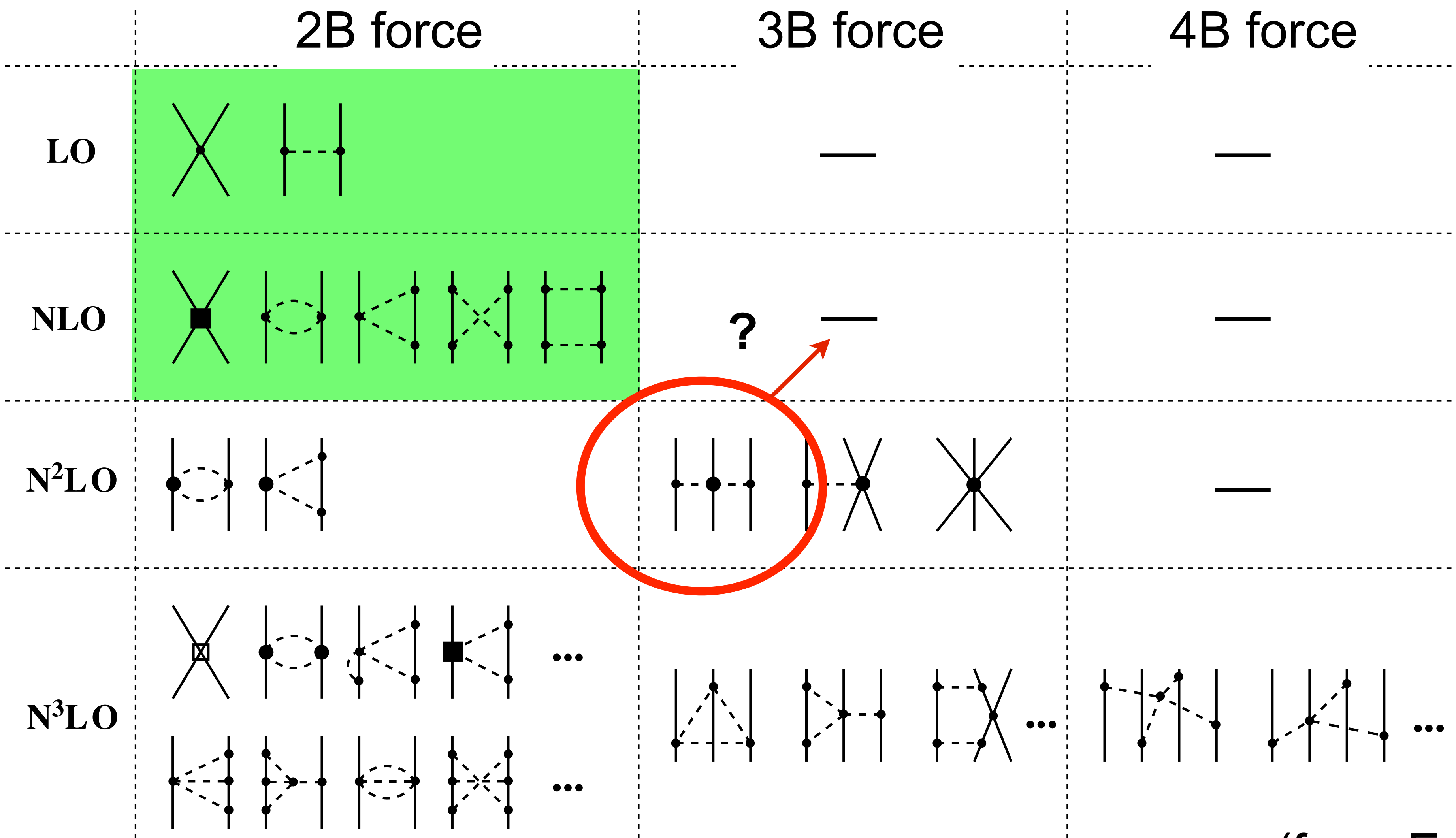
YN forces fail?
Is there any three-baryon force dependence as expected from ordinary nuclei?

The light hypernuclei provide non-trivial constraints on the YN interaction ...

... if we are able to control the 3BF contributions

Chiral (hyper-)nuclear interactions

non-perturbativity of A ≥ 2 requires to
perform chiral expansion for a potential which is used to solve a Schrödinger equation



5 NN/YN short range parameters

18 NN/YN short range parameters

3BF in N^2LO when decouplet neglected

(from Epelbaum, 2008)

Systematically improvable BB, 3B, 4B, ... interactions

Qualitatively: BB >> 3B >> 4B ...

In ordinary nuclei:

estimate accuracy using **cutoffs** of the Lippmann-Schwinger equation

Results: LO & NLO for $^{3}_{\Lambda}\mathrm{H}$

LO

Λ [MeV]	550	600	650	700	Jülich 04	Nijm SC97f	Nijm SC89	Expt.
E_Λ [MeV]	0.14	0.12	0.12	0.13	0.06	0.08	0.14	0.13(5)
P_Σ	0.2%	0.3%	0.4%	0.5%	0.2%	0.2%	0.5%	-
^{1}a [fm]	-1.9	-1.9	-1.9	-1.9	-2.6	-2.5	-2.6	?
^{3}a [fm]	-1.2	-1.2	-1.2	-1.2	-1.7	-1.7	-1.4	?

cutoff dependence of $E_\Lambda < 0.01$ MeV

NLO

Λ [MeV]	500	550	600	650	700
E_Λ [MeV]	0.12	0.10	0.08	0.09	0.10
P_Σ	0.2 %	0.3 %	0.2 %	0.2 %	0.2 %
^{1}a [fm]	-2.9	-2.9	-2.9	-2.9	-2.9
^{3}a [fm]	-1.6	-1.6	-1.6	-1.6	-1.6

cutoff dependence of $E_\Lambda < 0.02$ MeV

- cutoff dependence of separation energies indicates size of three-baryon forces
  → appears to be negligible here, **needs to be studied further**
- small scattering lengths in LO changes back to standard values in NLO
- Σ probability is generally small, largest values for old Nijm SC89

Three-*nucleon* force dependence

three-***nucleon*** forces are not included in the following calculations

first compare E_Λ including or not including 3NFs

example of EFT-NLO (Λ=600 MeV)

	w/o 3NF	w/ 3NF
E_Λ (0^+) [MeV]	1.47	1.38
E_Λ (1^+) [MeV]	0.71	0.65

(results for l_{sum}=6)

- dependence on 3NF is below 100 keV in both four-body states

  ➡ at this point 3NFs do not need to be included
- consistent with NN model dependence

Results: LO EFT for ${}^{4}_{\Lambda}\mathrm{H}$

Λ [MeV]	550	600	650	700	Jülich 04	Nijm SC97f	Nijm SC89	Expt.
$E_{sep}(0^+)$ [MeV]	2.6	2.5	2.4	2.4	1.8	1.6	1.8	2.04
$E_{sep}(1^+)$ [MeV]	1.9	1.5	1.2	1.0	2.4	0.5	0.0	1.00
ΔE_{sep} [MeV]	0.8	1.0	1.1	1.3	-0.52	0.99	1.8	1.04

→

$$E_\Lambda\left({}^4_\Lambda\mathrm{H};0^+\right) = (2.5 \pm 0.1)\ \mathrm{MeV}$$
$$E_\Lambda\left({}^4_\Lambda\mathrm{H};1^+\right) = (1.4 \pm 0.5)\ \mathrm{MeV}$$

- LO EFT results are (almost) consistent with experiment given the sizable cutoff dependence
- cutoff dependence for 0^+ state ?

- **higher order calculations required!**

Results: NLO EFT for ${}^4_\Lambda\mathrm{H}$

Λ [MeV]	500	550	600	650	700	Jülich 04	Nijm SC97f	Nijm SC89	Expt.
$E_\Lambda(0^+)$ [MeV]	1.62	1.52	1.47	1.52	1.61	1.8	1.6	1.8	2.04
$E_\Lambda(1^+)$ [MeV]	0.96	0.85	0.73	0.83	0.90	2.4	0.5	0.0	1.00
ΔE_Λ [MeV]	0.66	0.67	0.75	0.69	0.71	-0.52	0.99	1.8	1.04

→

LO

$$E_\Lambda\left({}^4_\Lambda\mathrm{H};0^+\right) = (2.5 \pm 0.1)\ \mathrm{MeV}$$
$$E_\Lambda\left({}^4_\Lambda\mathrm{H};1^+\right) = (1.4 \pm 0.5)\ \mathrm{MeV}$$

→

NLO

$$E_\Lambda\left({}^4_\Lambda\mathrm{H};0^+\right) = (1.55 \pm 0.08)\ \mathrm{MeV}$$
$$E_\Lambda\left({}^4_\Lambda\mathrm{H};1^+\right) = (0.85 \pm 0.12)\ \mathrm{MeV}$$

- LO/NLO results: indication that LO uncertainty in 0^+ is underestimated
- NLO results in line with model results
- cutoff dependence at NLO consistently 100 keV
  → **indicates** 3BF force contribution is small
- **but:** NLO results are **inconsistent** with experiment

cutoff variation **indicates** a small 3BF contribution

deviation from experiment:

short distance SU(3) breaking?

accuracy of data?

CSB at NLO & for model interactions

0^+ state

Λ [MeV]	500	550	600	650	700	Jülich 04	Nijm SC97e	Nijm SC89	Expt.
ΔT [keV]	48	56	55	51	45	0	47	132	-
ΔV_{NN} [keV]	-2	2	2	0	-3	-31	-9	-9	-
ΔV_{YN} [keV]	-11	-12	-11	-9	-8	2	37	228	-
tot [keV]	35	46	46	42	34	-29	75	351	350
$P_{\Sigma-}$	1.1%	1.2%	1.2%	1.1%	1.0%	0.3%	1.0%	2.7%	-
$P_{\Sigma0}$	0.6%	0.7%	0.7%	0.6%	0.5%	0.3%	0.5%	1.4%	-
$P_{\Sigma+}$	0.1%	0.1%	0.2%	0.1%	0.1%	0.3%	0.0%	0.1%	-

- **kinetic energy contribution is driven by Σ component**
- NN force contribution due to small deviation of Coulomb
- YN force contribution:
  - *SC89 CSB is strong*
  - *NLO CSB is zero, only Coulomb acts (Σ component)*

CSB at NLO & for model interactions

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1^+ state

Λ [MeV]	500	550	600	650	700	Jülich 04	Expt.
ΔT [keV]	14	17	16	15	12	15	-
ΔV_{NN} [keV]	-5	-1	1	-2	-4	-43	-
ΔV_{YN} [keV]	-5	-5	-5	-5	-4	-6	-
tot [keV]	4	11	12	8	4	-34	240
$P_{\Sigma-}$	0.8%	0.8%	0.8%	0.7%	0.7%	0.5%	-
$P_{\Sigma 0}$	0.6%	0.7%	0.6%	0.6%	0.5%	0.3%	-
$P_{\Sigma+}$	0.5%	0.5%	0.4%	0.4%	0.4%	0.1%	-

- **kinetic energy contribution is driven by Σ component**
- NN force contribution due to small deviation of Coulomb
- YN force contribution:
  - *SC89 CSB is strong*
  - *NLO CSB is zero, only Coulomb acts (Σ component)*

CSB is linked to Σ component

deviation from experiment:

can we increase Σ component?

accuracy of data?

Conclusions

- reliable A=4 calculations exist
  - *including Λ-Σ conversion is important*

- YN interactions are not understood
  - *YN models fail (need to study ESC models)*
  - *NLO of chiral EFT: freedom to adjust YN forces?*

- hypernuclei are an essential source of information on YN
  - *it is not trivial to describe the simplest systems consistently*
  - *J-PARC & MAMI experiments for very light hypernuclei are important*

- CSB for four-body hypernuclei is a puzzle
  - *related to Λ-Σ conversion*
  - *J-PARC & MAMI experiments for very light hypernuclei are important*

Outlook

- Issue 1: **Three-baryon forces**
  - *we need to control the three-baryon forces to constrain YN interactions*
  - *naively cutoff variation estimates size of all 3BF terms* ***(true?)***
  - *smallest cutoff used (500 MeV) > m_{Σ^*}-m_Λ ≈ 270 MeV*
    - *smaller cutoffs using SRG evolved YN interaction* ***(in progress)***
  - *cutoff variation estimates contact terms*

    ↔ power counting indicates leading contributions are long-ranged
    - *consider decouplet terms* ***(possible?, see Gal, Soper, Dalitz(1971)?)***
- Issue 2: **Consequences of adjustments of the YN forces**
  - *explore flavor-SU(3) breaking* ***(agreement with data possible?)***
  - *larger Σ component in hypernuclei* ***(how to adjust YN interaction?)***
  - *leading CSB contributions* ***(Nijmegen SC97 approach?)***
- Issue 3: **Larger systems**
  - *NCSM in Jacobi basis in preparation for ordinary nuclei* ***(Susanna Liebig)***
  - *A=5, 6, ... will be interesting*